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THESIS

PARAMETER PLANE DESIGN OF ELECTRICAL FILTERS CONTAINING NONLINEAR ELEMENTS

by

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# Parameter Plane Design of Electrical Filters Containing Nonlinear Elements

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ABSTRACT

A parameter-plane method, based upon a quasi-frozen assumption, for the design of electrical filters containing slowly varying nonlinear elements is presented together with examples of its application to the lowpass, highpass, bandpass and band-reject filter types. The nonlinear magnitude and frequency characteristics are presented in the form of parameter-plane graphs which allow value judgements to be made concerning the choice of nonlinear elements used in the filter design. The computer programs used to generate the graphs are also included.

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# I. INTRODUCTION

A filter is a frequency-sensitive unit which discriminates between desired and undesired frequencies. Ideally the filter would have a unity-gain passband and a zero-gain stopband with no transition region between the bands. Also, its phase shift would be linear within the passband. Such filters do not exist today. The ideal filter is the optimum case and serves as a standard of comparison for all other filters. Filter design usually stresses either the magnitude response or the phase response depending on the intended use of the filter.

Filters may be grouped into four distinct types based on their frequency discrimination: lowpass, highpass, bandpass, and band-reject. While classical design used passive, lumped, linear elements and image-parameter methods to realize these filter types, modern design uses the polynomial method of network synthesis and considers all types of elements in both their active and passive forms. The addition of nonlinear elements in these filters allows a closer approach to the ideal response than possible if the elements were linear. At the same time however, the design problem becomes more complicated.

A parameter-plane method for the design of filter circuits is presented in this thesis. The method provides a relatively simple, easy-to-use way of incorporating

nonlinear elements into the design of electrical filter circuits.

# II. THE PARAMETER PLANE METHOD

The mathematical analysis of a linear dynamic system usually evolves into the evaluation of the roots of a polynomial formed from the differential equation description of the system. The root locations of this characteristic equation are then interpreted with respect to the system response characteristics. Any change in any circuit parameter value, however, changes the root locations. Some changes will improve the circuit response while others will degrade it.

Mitrovic, Thaler, Siljak and others developed analysis methods which consider such parameter value variations [1]. The original method of Mitrovic was rather limited since only the last two coefficients of the characteristic equation were allowed to vary. Lines of constant damping ratio ($\zeta$) were mapped from the s-plane into the plane of the variable coefficients using the characteristic equation as the mapping function. These transformed curves were then used to determine the variable coefficient values required to place the characteristic equation roots at desired locations.

Generalization and extension of Mitrovic's coefficient plane by Siljak and Hollister [2] led to the concept of the $\alpha,\beta$ parameter plane. This method considers the variable coefficients of the characteristic equation as nonlinear functions of two system parameters, $\alpha$ and $\beta$.

$$F(s) = \sum_{i=0}^{n} a_i s^i = 0, \; a_i = f_i(\alpha,\beta).$$

Curve families of constant damping ratio and undamped natural frequency are commonly mapped into the $\alpha,\beta$ parameter plane and used to infer system characteristics.

In the case of a system transfer function, however, the characteristic equation is formed from the denominator. The numerator is not included in the formulation. Therefore, full frequency-response information cannot be gained from using the characteristic equation in the parameter plane method. Thaler and Thompson developed a form of the transfer function which could be used in the parameter-plane method [3]. This transfer function form has been used by Glavis [4] and Staples [5] to make frequency-response plots and analyses of systems in which the coefficients contained linear combinations of $\alpha$ and $\beta$.

The derivation of the transfer function form developed in Ref. 3 follows:

Let the system transfer function be given as

$$T(s) = \frac{N(s)}{D(s)}$$

where $N(s)$ and $D(s)$ are each of the form

$$N(s) = \sum_{i=0}^{n} a_i s^i$$

and the terms are nonlinear function of two parameters, $\alpha$ and $\beta$. The squared magnitude function can be obtained from

$$m^2 = T(j\omega)\cdot T(-j\omega).$$

If the polynomials N(s) and D(s) are separated into their even and odd parts,

$$T(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)},$$

$$T(-s) = \frac{N_e(s) - N_o(s)}{D_e(s) - D_o(s)}$$

and $$T(s)\cdot T(-s) = \frac{N_e(s)^2 - N_o(s)^2}{D_e(s)^2 - D_o(s)^2}.$$

From this it may be seen that both the numerator and denominator are even polynomials. When $s = j\omega$ both numerator and denominator are even in $\omega$. Thus

$$m^2 = \frac{\sum_{i=0}^{\ell} A_i \omega^{2i}}{\sum_{k=0}^{n} B_k \omega^{2k}}$$

where $A_i = f_i(1,\alpha,\beta,\alpha\beta,\alpha^2,\beta^2,\alpha^2\beta,\alpha\beta^2,\ldots)$

and similarly for the $B_k$ terms.

There are now four parameters present, alpha, beta, magnitude and omega. Any one may be expressed in terms of

the others and curves drawn depicting the various relationships among the parameters as defined by the transfer function. Phase versus omega may also be examined by rationalizing $T(j\omega)$ and using the arctan relationship to solve for the angle.

Although the original motivation for the parameter plane was to analyze linear dynamic systems, the method may also be used to analyze nonlinear systems. Consider a linear system. One of its characteristics is that a sinusoidal input signal of a given frequency will be repeated throughout the system as a sinusoidal signal of the same frequency. This is not always true in a circuit containing nonlinear elements. In most cases though, the internal signals are at least periodic and have the same period as the input signal. These signals may therefore be represented by a Fourier series. Thus a frequency-response transfer function may be defined relating the fundamental component of the output signal to the input signal and may be used in the parameter-plane analysis of a nonlinear system. The nonlinearity is considered a parameter whose characteristic is defined by a locus of points in a parameter plane e.g., alpha versus transfer-function magnitude. The function $\alpha(v)$ is assumed to vary slowly enough that it may be considered as "frozen" or fixed over the frequency range of interest.

Further, this parameter plane analysis logic may be reversed and the method used as a tool in the design of a

system. Thus a desired system response may be specified and the $\alpha,\beta$ parameter variations necessary to produce this response determined by the method.

The parameter-plane method would be far too laborious if done by hand. Therefore, FORTRAN IV computer programs have been developed by the author which allow the frequency-response form of the parameter plane method to be used in the design of filters with nonlinear elements. The program details are contained in Appendix A.

## III. DESIGN OF FILTERS WITH NONLINEAR ELEMENTS

Filters with nonlinear elements may be designed by reference to parameter-plane plots of the voltage transfer function of the filter. The design may emphasize the magnitude response, the phase response or both. The nonlinear elements are represented as parameters alpha and/or beta and used in conjunction with the parameters of magnitude and of normalized radian frequency, omega. A computer generates selected plots of any one parameter versus another with a third parameter held constant and the fourth incremented to form a family of curves. A fifth parameter, phase, may be plotted versus omega for family parameters alpha and beta. The desired filter response is noted on these plots and the required characteristics of a particular nonlinear element determined.

From given specifications a particular type of filter circuit is chosen and the transfer function determined. For simplicity let the nonlinear element be represented by the alpha parameter and be a function of output magnitude. Also assume that in this case the magnitude response is important while the phase response is not.

The computer is told to make a Bode plot of the linear response of the filter for each of a family of alpha values and the response required by the specifications is overplotted by hand onto the same graph (see Figure 1). Each

alpha-omega coordinate point along the specification curve indicates what the value of alpha must be in order to satisfy the specifications. These coordinate points are then picked off and transferred by hand to a computer-generated parameter-plane graph of alpha versus magnitude where beta has been held constant and omega incremented to form the curve family (see Figure 2). The locus of hand-plotted points defines the required characteristics of the particular nonlinear element with respect to the circuit output magnitude. If the nonlinear element were a function of frequency, the coordinate points would have been hand transferred to a computer-generated parameter-plane graph of alpha versus omega with magnitude as the family parameter. This would determine the necessary frequency characteristics of the nonlinear element.

Filters with two or more nonlinear elements may require plotting points in the alpha-beta parameter plane (magnitude held constant and omega incremented or vice versa) to determine possible value trade-offs between the nonlinear elements. The curve plotted on the graph of parameter versus magnitude or omega would still be used to define the nonlinear element characteristics. These same procedures could be followed with the response emphasis on phase rather than on magnitude.

An additional consideration is necessary. The transfer function is the ratio of one voltage to another, E2/E1. Consequently, the plotted response is that of E2 with E1

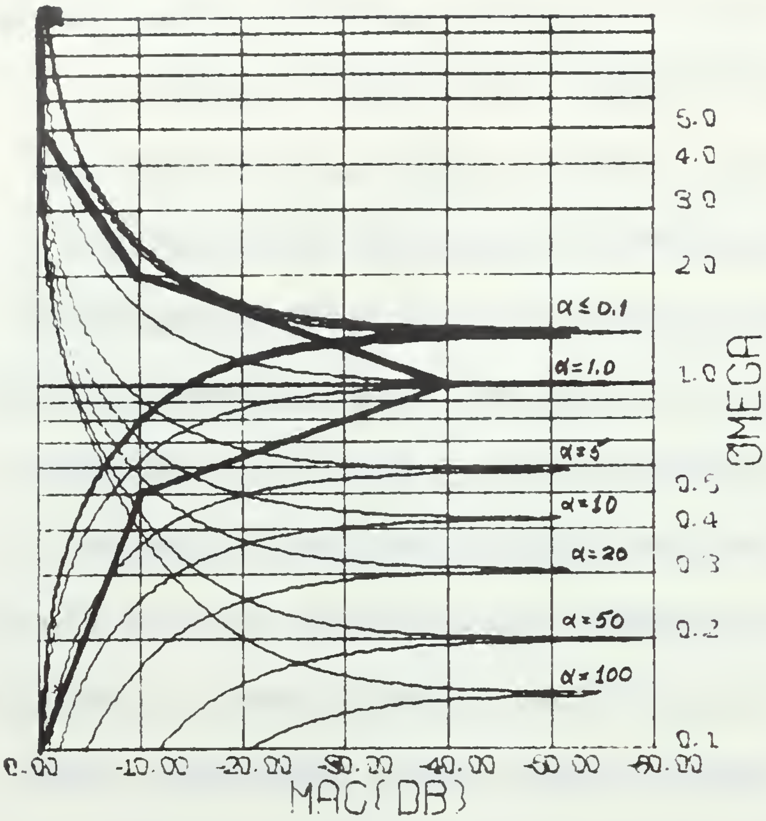


FIGURE 1.
BODE PLOT WITH
SPECIFICATION CURVE.

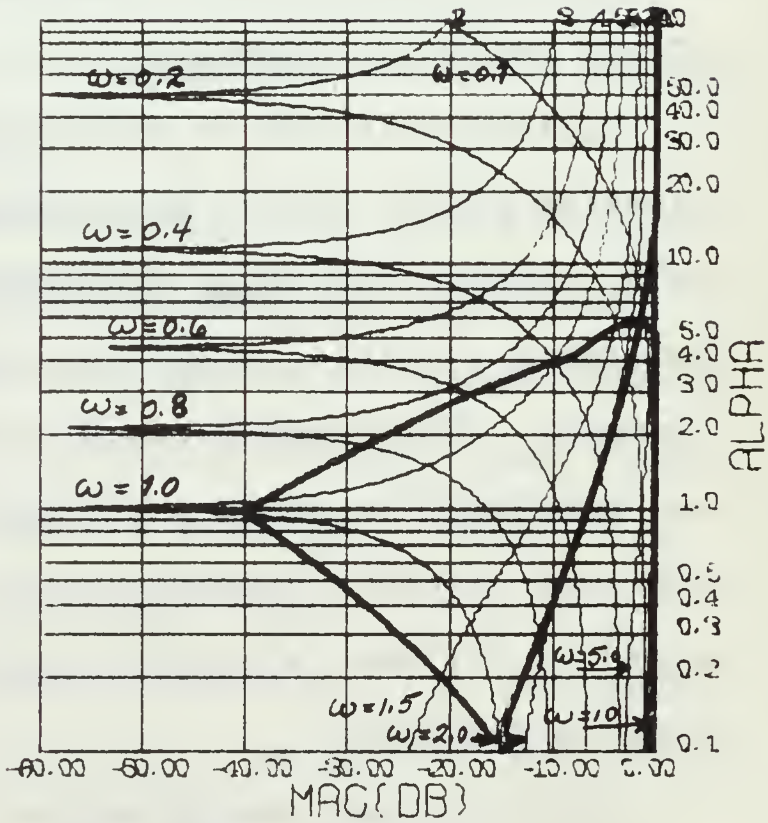


FIGURE 2.
PARAMETER PLANE PLOT
ALPHA VS MAGNITUDE.

set to one volt. Thus in realizing the filter design the nonlinear element must be made a function of E2/E1 and not a function of E2 alone. A circuit to perform this division may be omitted if the nonlinear element is a function of frequency or when simulation sensitivity checks indicate that the filter response will be within acceptable limits for the predicted range of input values.

Although the filters considered here are shown in a lumped R - L - C configuration, the R, L and C should be looked upon as representing voltage-current relationships rather than actual physical components. The lumped R - L - C could be transformed into distributed values and/or L - C interchanges made through the use of active networks such as gyrators, mutators, rotators, etc. Time invariance is assumed however.

The steps in the design of a filter are:

1. Form the transfer function, T(s), of the desired filter type designating the nonlinear elements as alpha, beta.

2. Determine which computer-generated plots will be needed.

3. Encode data and use the computer programs described in the appendix to generate the plots.

4. Select the Bode and/or phase plot as appropriate and draw in by hand the specified response curve.

5. Hand transfer the coordinate points along the response curve to the computer-generated parameter-versus-magnitude or parameter-versus-omega graphs.

6. Determine the characteristics of the nonlinear element from the locus of points plotted in step five above.

7. Check the design by computer simulation using several voltage input values to determine the effect of a changing input.

8. If the nonlinear element is a function of magnitude, decide whether or not the divider circuit is necessary.

9. Frequency shift the component values from the normalized radian frequency to the frequency of interest. Impedance shifting may also be used.

In the case of two or more nonlinear elements, step five will include the alpha-versus-beta plot for possible parameter value trade-offs.

## A. FILTERS WITH ONE NONLINEAR ELEMENT

The following filter examples cover the four frequency types, lowpass, highpass, bandpass and band-reject. Each of these basic filters is made up of three elements, one R, one L and one C with the nonlinear element being a function of the output magnitude. The R element has been chosen as the nonlinear element since it may be readily associated with a field-effect transistor. The L or the C could just as well have been chosen.

1. Lowpass Filter

Figure 3 illustrates one possible lowpass filter configuration. Since frequency scaling will be employed as the last design step, the L and C values are normalized to one. The parameter alpha replaces the nonlinear element R. Thus the circuit transfer function is

$$T(s) = \frac{1}{1 + \alpha s + s^2} .$$

The necessary computer plots for the design of this circuit are the magnitude-versus-omega (Bode) and alpha-versus-magnitude plots.

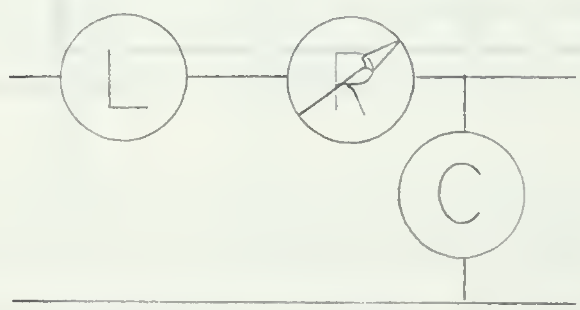


Figure 3. Lowpass Filter

Figure 4 is the Bode plot for the circuit. The alpha parameter curves on the plot indicate the output magnitude of the filter for the normalized radian frequency range of 0.1 to 10.0 radians when alpha is fixed at a particular parameter value. Thus if alpha were set equal to 1.0, the output-versus-frequency response of the filter would be that indicated by the curve for alpha equal to 1.0. Therefore, this graph illustrates the response of a linear filter

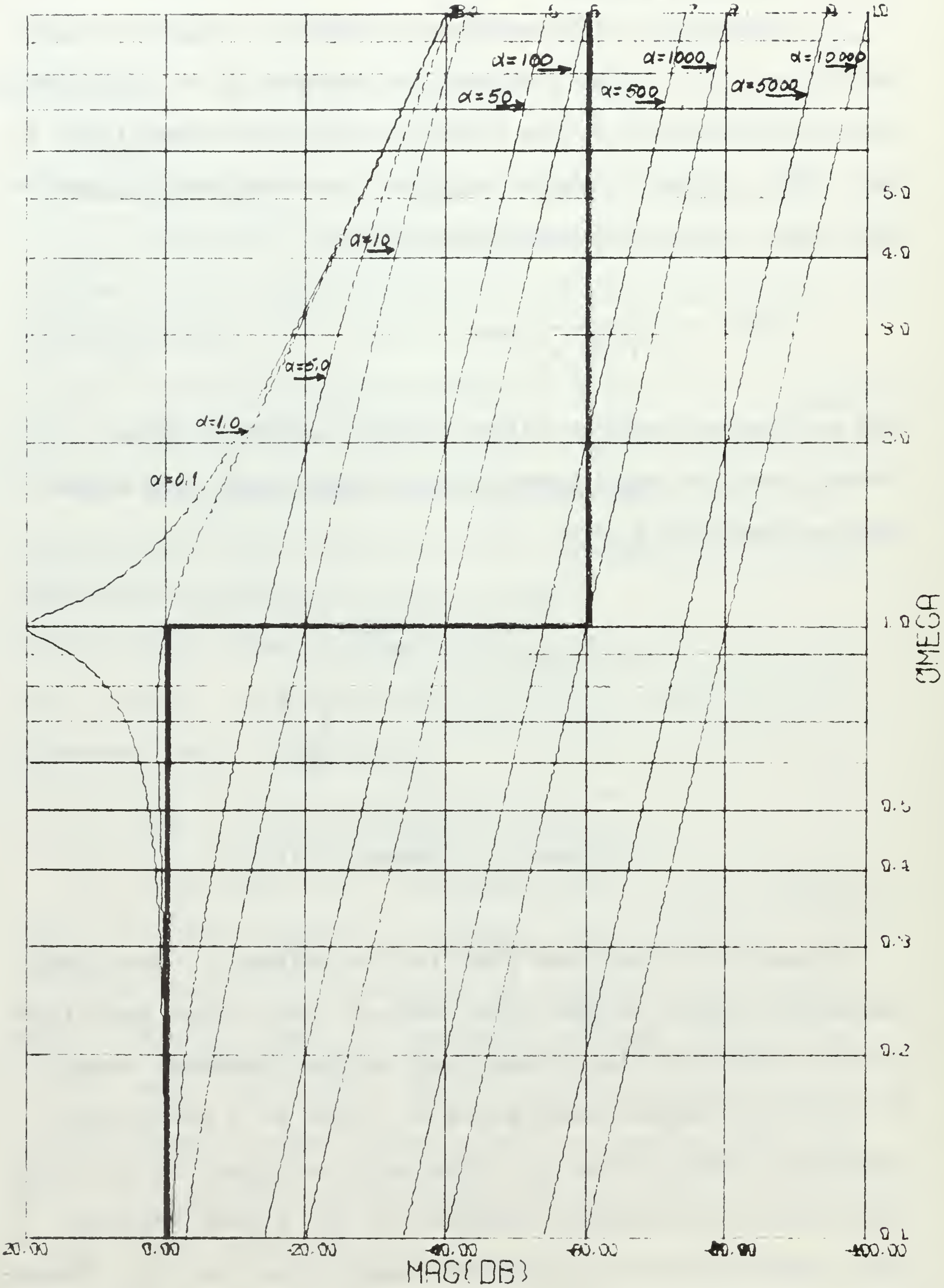


FIGURE 4.
R.A.NICHOLS, BODE PLOT. PARAMETER=ALPHA
LOWPASS FILTER, L=C=1.0, R=ALPHA

for several values of one of its elements, R. The filter specifications are indicated by the heavy line drawn on the graph by hand. In this case it is assumed to be the magnitude response of an ideal filter. The zero gain in the stopband of the ideal filter is represented by a line at the minus 60.0-dB level.

The coordinate points of the intersection of the alpha parameter curves and the desired response line in Figure 4 are picked off and transferred by hand to the alpha-versus-magnitude parameter-plane plot in step five of the design procedure. This plot is shown in Figure 5. The heavy line is the locus of points transferred. It indicates the desired characteristics of the nonlinear element when the value of the element is a function of the filter output magnitude.

The initial value of alpha for this plot is less than or equal to 2.0 at an omega of 0.1. The required value of alpha at cutoff is 1.0. For a flat, zero-dB response between these two omega points, required alpha values are 1.5 at an omega of 0.2 and 1.35 at an omega equal to 0.5. The plot also indicates that an alpha value of 1.0 would deviate from the ideal response by +1.0 dB at omega equal to 0.5 and less than about +1.2 dB between 0.5 and cutoff. If this is tolerable, alpha may be fixed at 1.0 for values of omega less than 1.0 and the design problem simplified.

In the region where omega is equal to 1.0 the alpha-magnitude relationship is logarithmic. The filter output

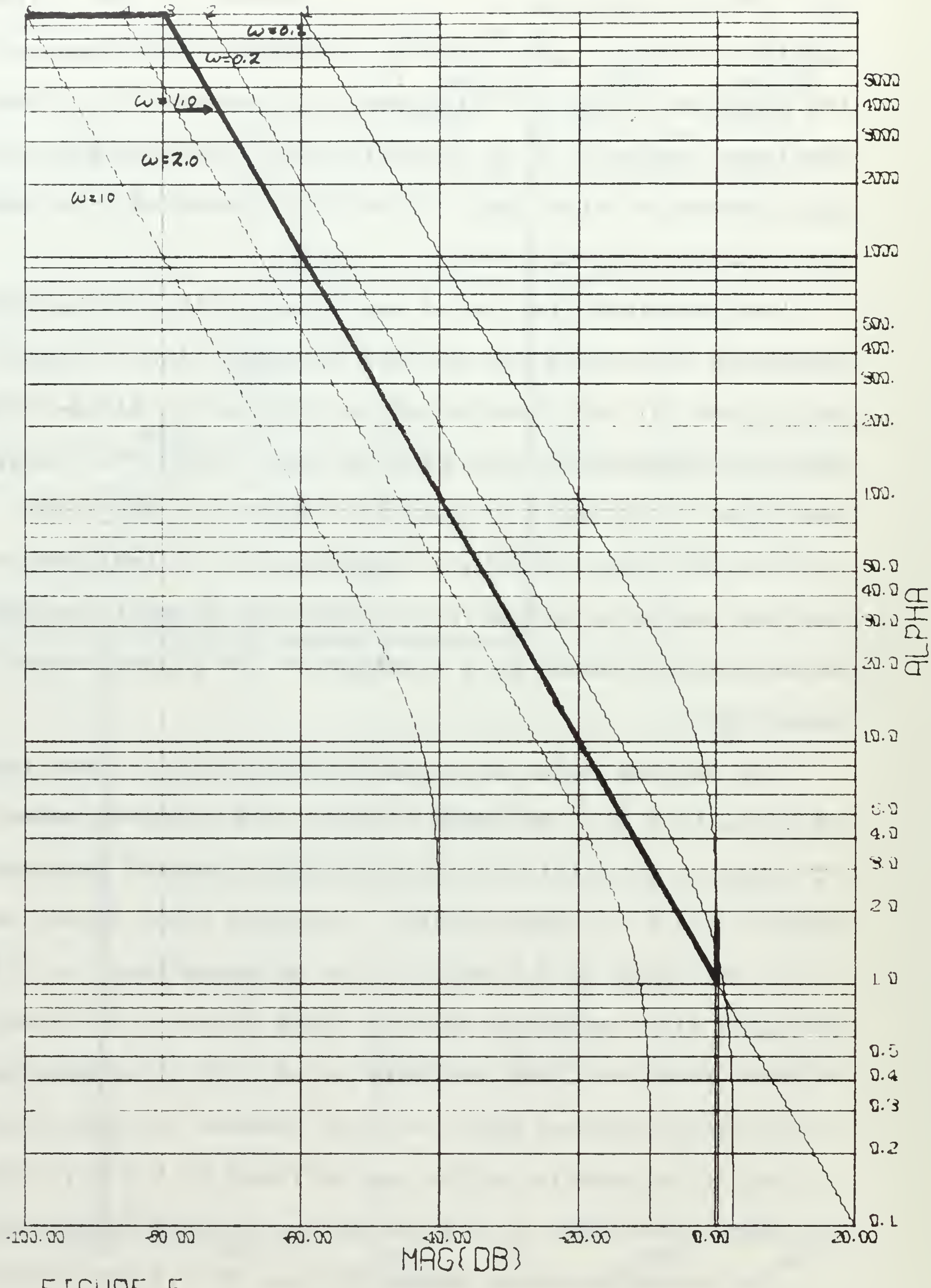


FIGURE 5.
R.A.NICHOLS.ALPHA VS MAGNITUDE(DB).PARAM=OMEGA
LOWPASS FILTER. L=C=1.0. R=ALPHA

is 0.0 dB at alpha equal to 1.0, -60.0 dB at alpha equal to 1000 and -80.0 dB at an alpha of 10,000. The minus 60.0-dB level requires alpha to have a three-decade range in value. The minus 80.0-dB level is attainable with the four-decade range of field-effect transistors. Alpha may remain at this last value for frequencies above 1.0. If this is so, the output level will fall off at a 20-dB/decade rate.

At this point consideration should be given to the realization of the circuit. Although theoretically any function may be generated, the economics of the situation usually limits the degree of circuit complexity allowed and causes simplifying approximations to be investigated. The fixing of alpha at 1.0 for the frequency region below 1.0 and to a fixed value for the region above 1.0 would be simplifications in the case of the lowpass filter. The drop in output magnitude below 0.0 dB at cutoff when alpha is equal to 1.0 may be used to initiate the required parameter value change. The change in R could come from the filter transfer-function output voltage being subtracted from a fixed reference level and the difference applied to the gate of a field-effect transistor.

Figure 6 is the phase-plot companion to the Bode plot of Figure 4. The dark line indicates the expected phase response of the filter when alpha equals 1.0 at values of omega below 1.0 and equals 10,000 at omega values above 1.0.

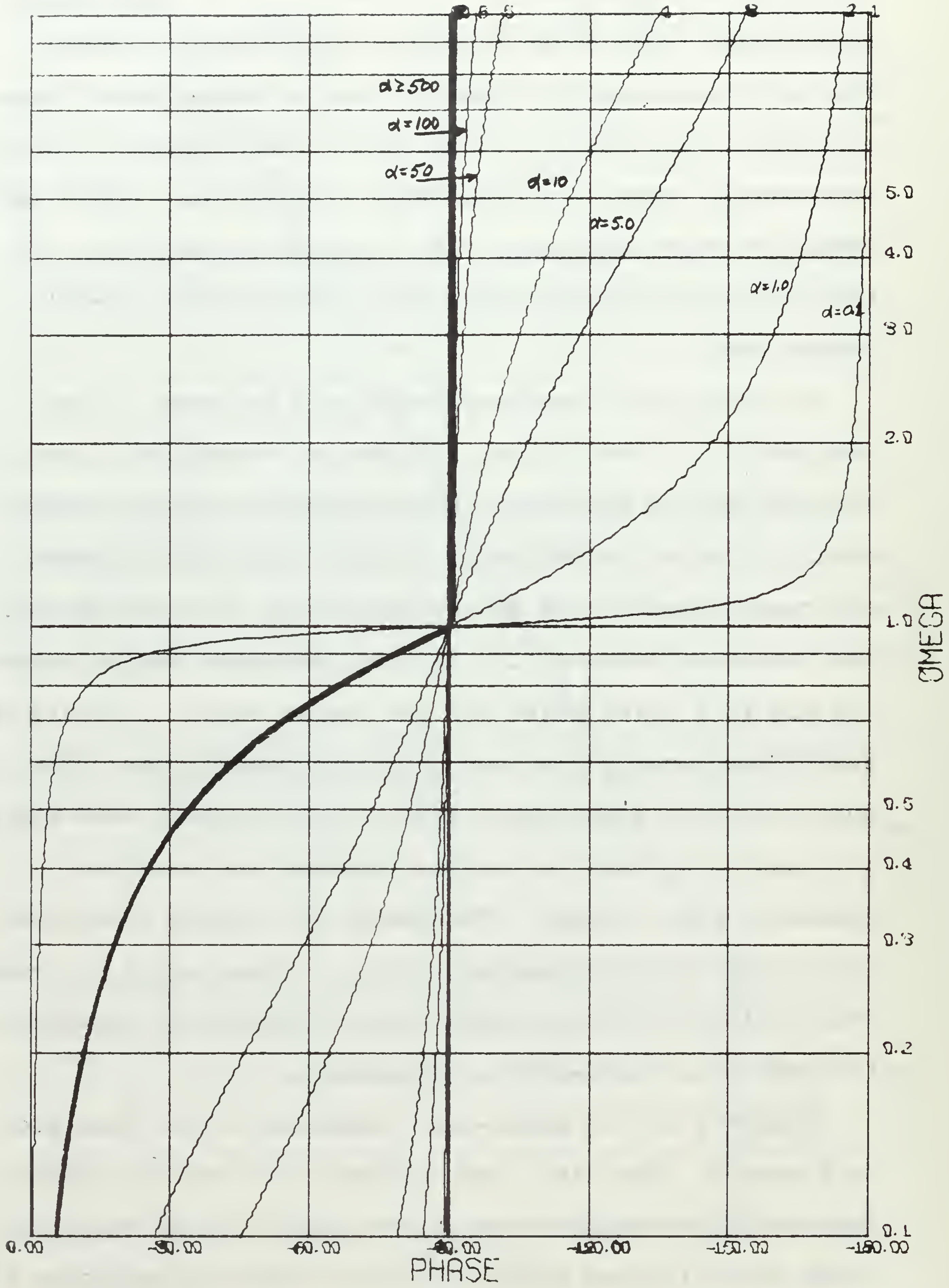


FIGURE 6.
R.A.NICHOLS, PHASE PLOT, PARAMETER=ALPHA
LOWPASS FILTER, L=C=1.0, R=ALPHA

The phase fall-off in the latter region is very slow. Designing for the ideal magnitude response does not yield a linear phase filter.

If alpha were a function of frequency, the plot of alpha versus omega, Figure 7, would replace the alpha-versus-magnitude plot. Again, the heavy line is the locus of points transferred from the Bode plot of Figure 4. The 0.0-dB curve defines the exact values of alpha necessary for a completely flat magnitude response in the passband region. The area between the 0.0 and +1.0 dB curves indicates the latitude in alpha values available for frequencies below 0.2 radians.

The computer simulation and input sensitivity check results are depicted in Figure 8 for the output magnitude and in Figure 9 for the phase. A 1.0-dB region of uncertainty or lag is assumed for the output level at which the parameter value change is initiated. Alpha is fixed at 1.0 for output values above the minus 1.0-dB level and allowed to go as far as 10,000 whenever the output drops below -1.0 dB. The curves represent the circuit response when the divider circuit is omitted and the input voltage varies around 1.0.

2. <u>Highpass Filter</u>

Figure 10 illustrates a possible highpass filter configuration. As in the lowpass filter circuit L and C are normalized to one and alpha represents the nonlinear R in the transfer function

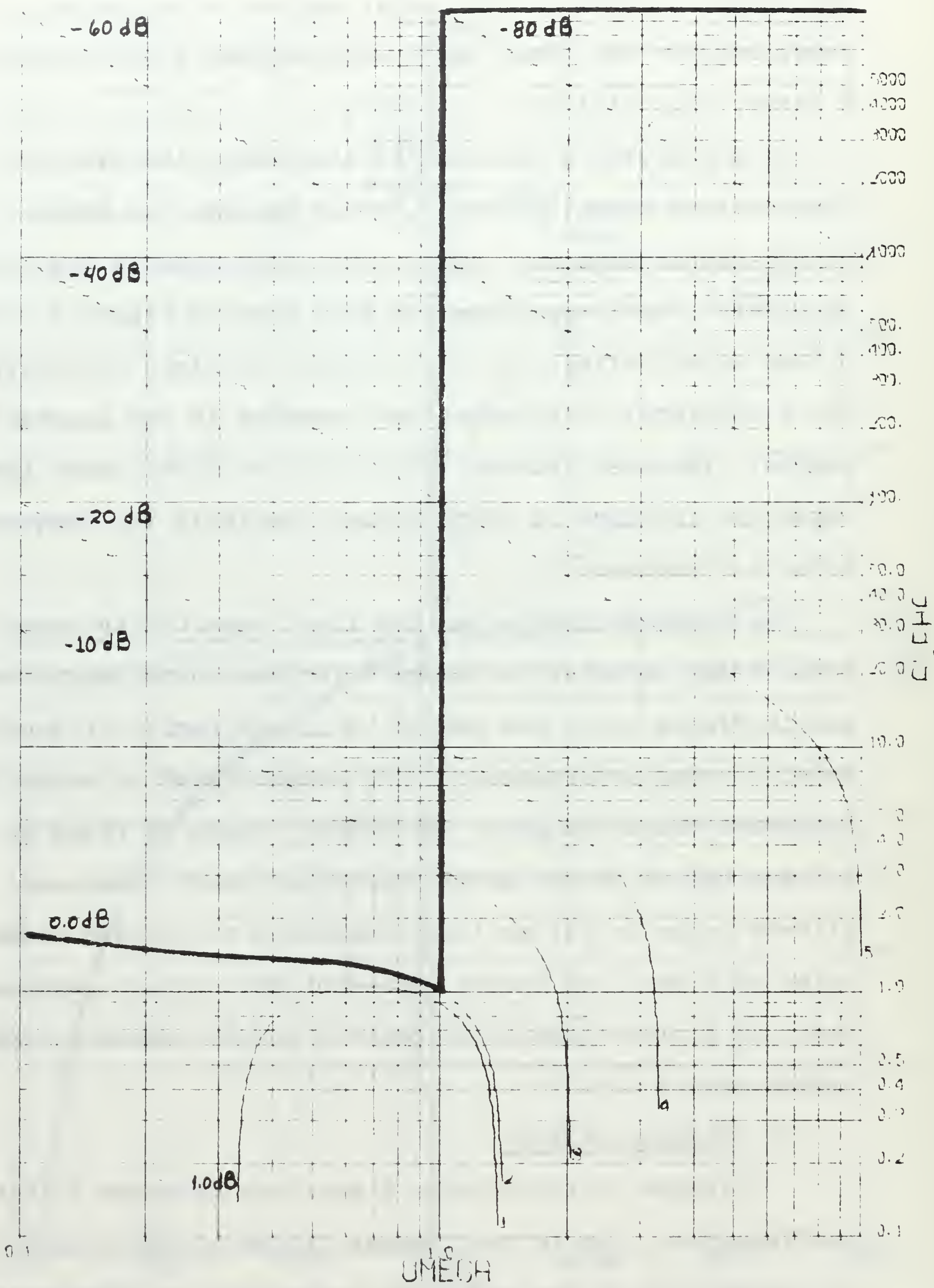


FIGURE 7.
R.A.NICHOLS, ALPHA VS OMEGA, PARAMETER=MAG(DB)
LOWPASS FILTER, L=C=1.0, R=ALPHA

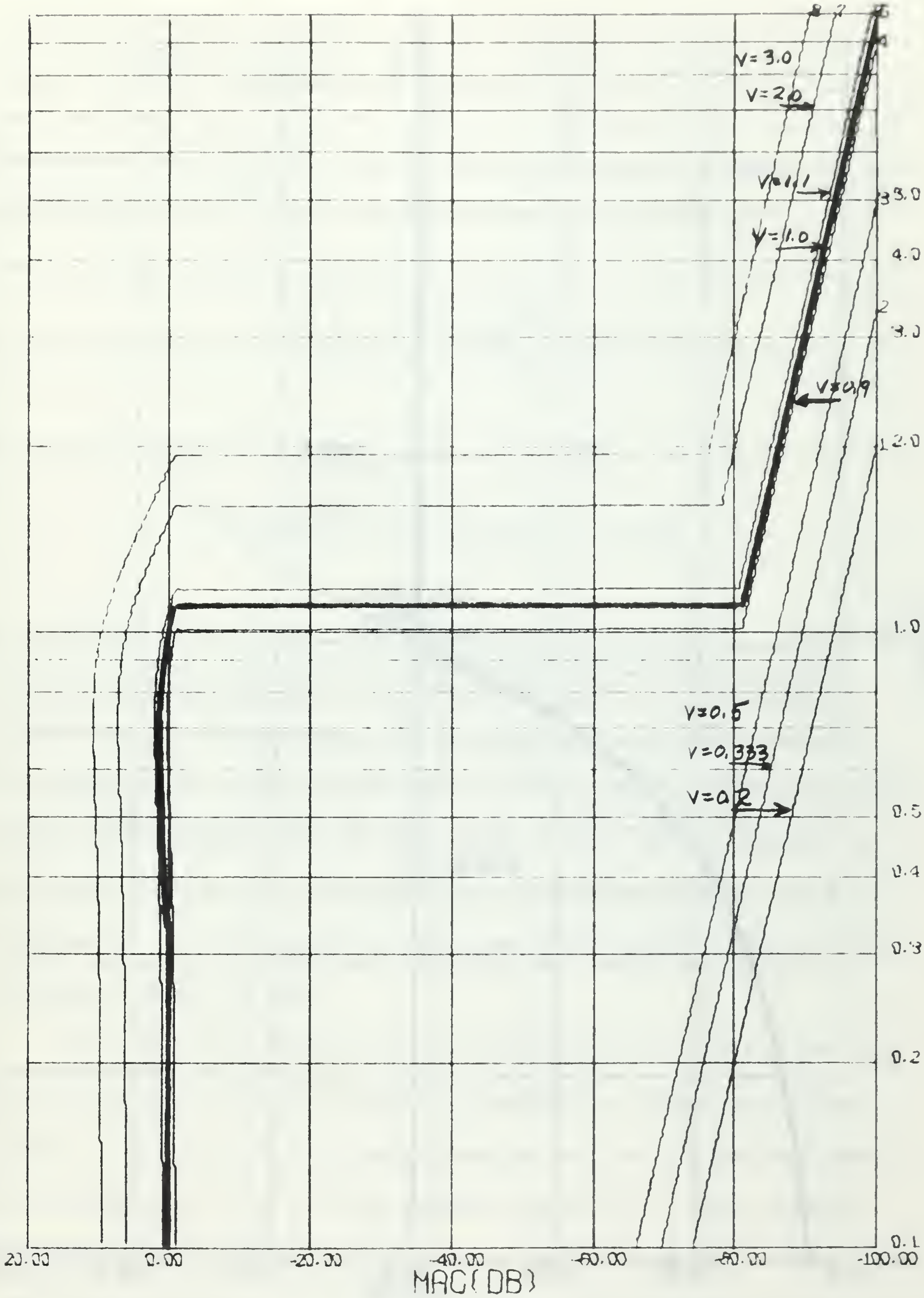


FIGURE 8.
R.A.NICHOLS. BODE PLOT. PARAMETER=(VINPUT*1.0).
LOWPASS FILTER. L=C=1.0. R=ALPHA

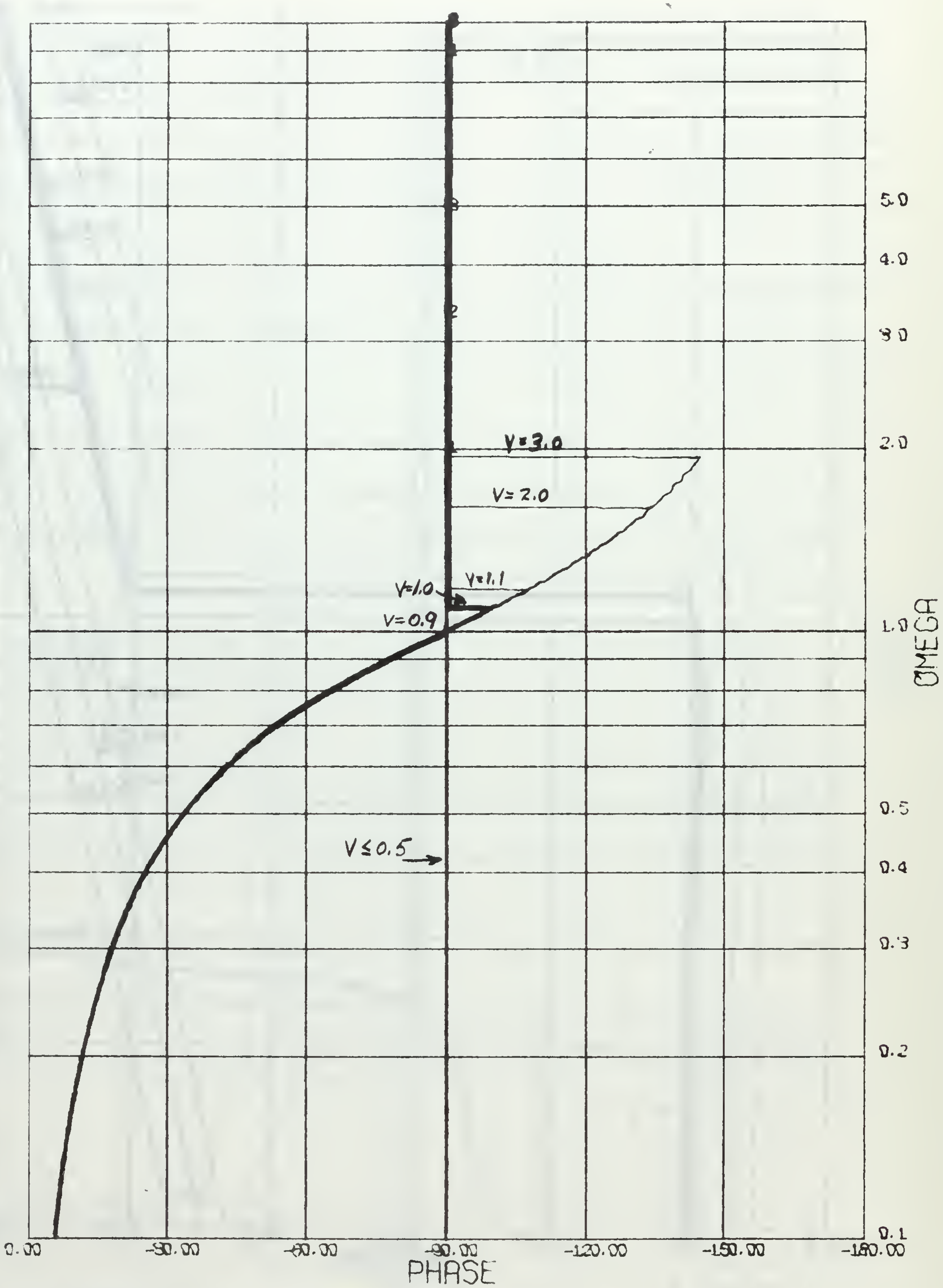


FIGURE 9.
R.A.NICHOLS, PHASE PLOT, PARAMETER=(VINPUT*1.0).
LOWPASS FILTER. L=C=1.0, R=ALPHA

$$T(s) = \frac{s^2}{1 + \alpha s + s^2} .$$

The necessary computer plots for the design of this filter are the Bode and the alpha-versus-magnitude plots

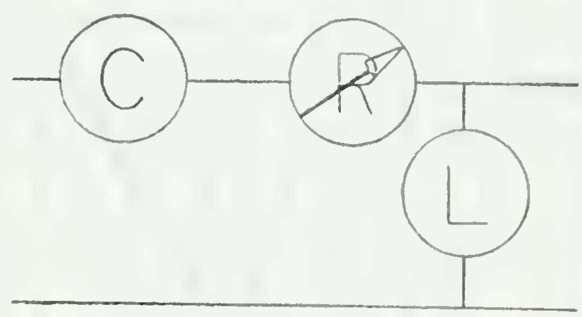


Figure 10. Highpass Filter

Figure 11 is the Bode plot for this circuit. Since the circuit configuration is related to that of the lowpass filter by an interchange of frequency-sensitive elements, the Bode plot is a reflection of the lowpass filter circuit about the omega equal to 1.0 axis. This relationship is further brought out by the alpha-versus-magnitude plot of Figure 12. It is exactly the same as Figure 5 except for reversed omega values.

Circuit realization is also related to that of the lowpass filter. In this case the parameter alpha could be fixed at 10,000 in the omega region below 1.0 and allowed to decrease to 1.0 in the region above 1.0. The filter transfer-function output would be added to a reference level and the sum applied to the gate of a field-effect transistor. The change in alpha would be initiated when the output magnitude exceeded a given value, say -80.0 dB.

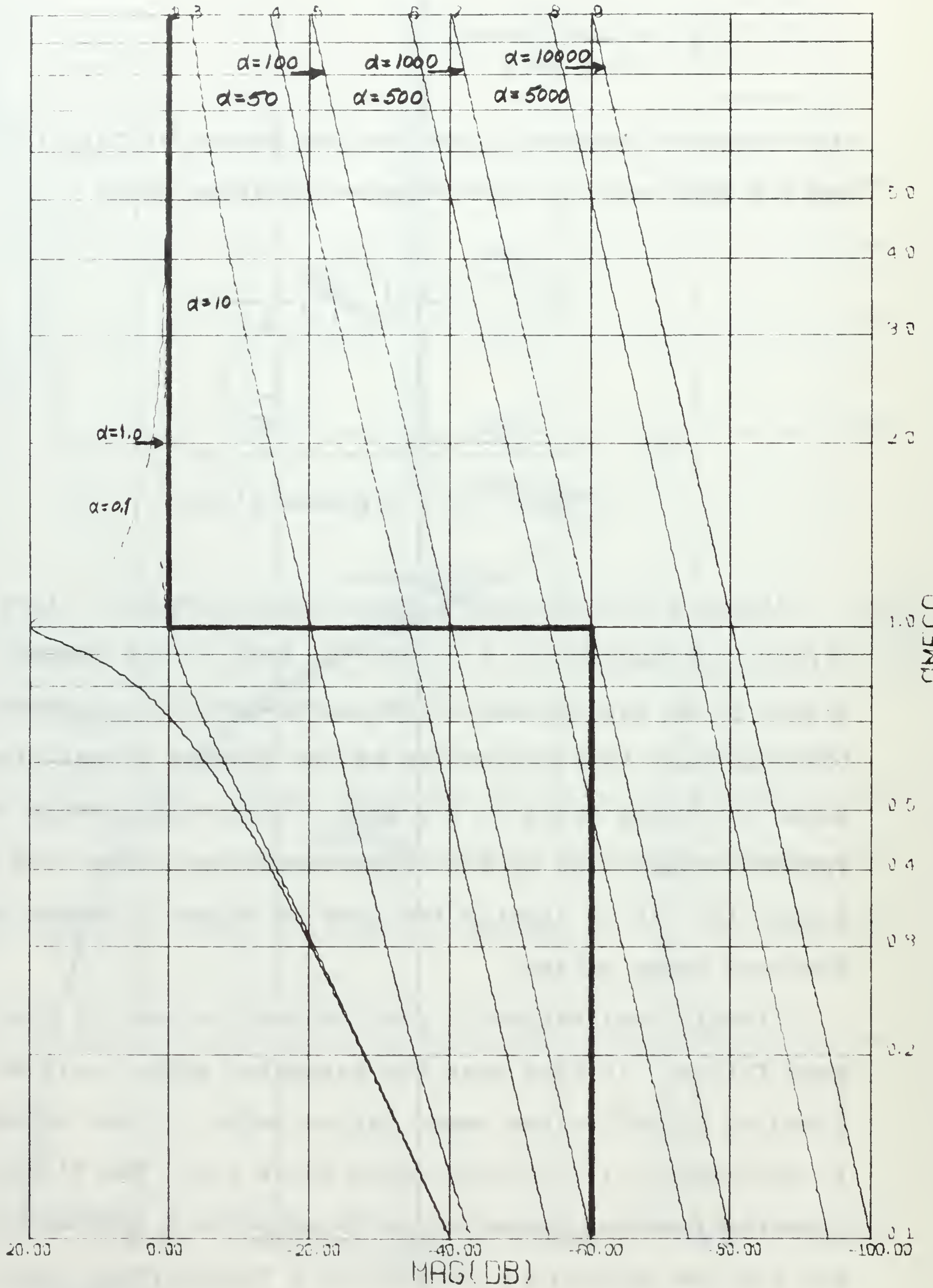


FIGURE 11.
R.A.NICHOLS, BODE PLOT, PARAMETER=ALPHA
HIGHPASS FILTER, L=C=1.0, R=ALPHA

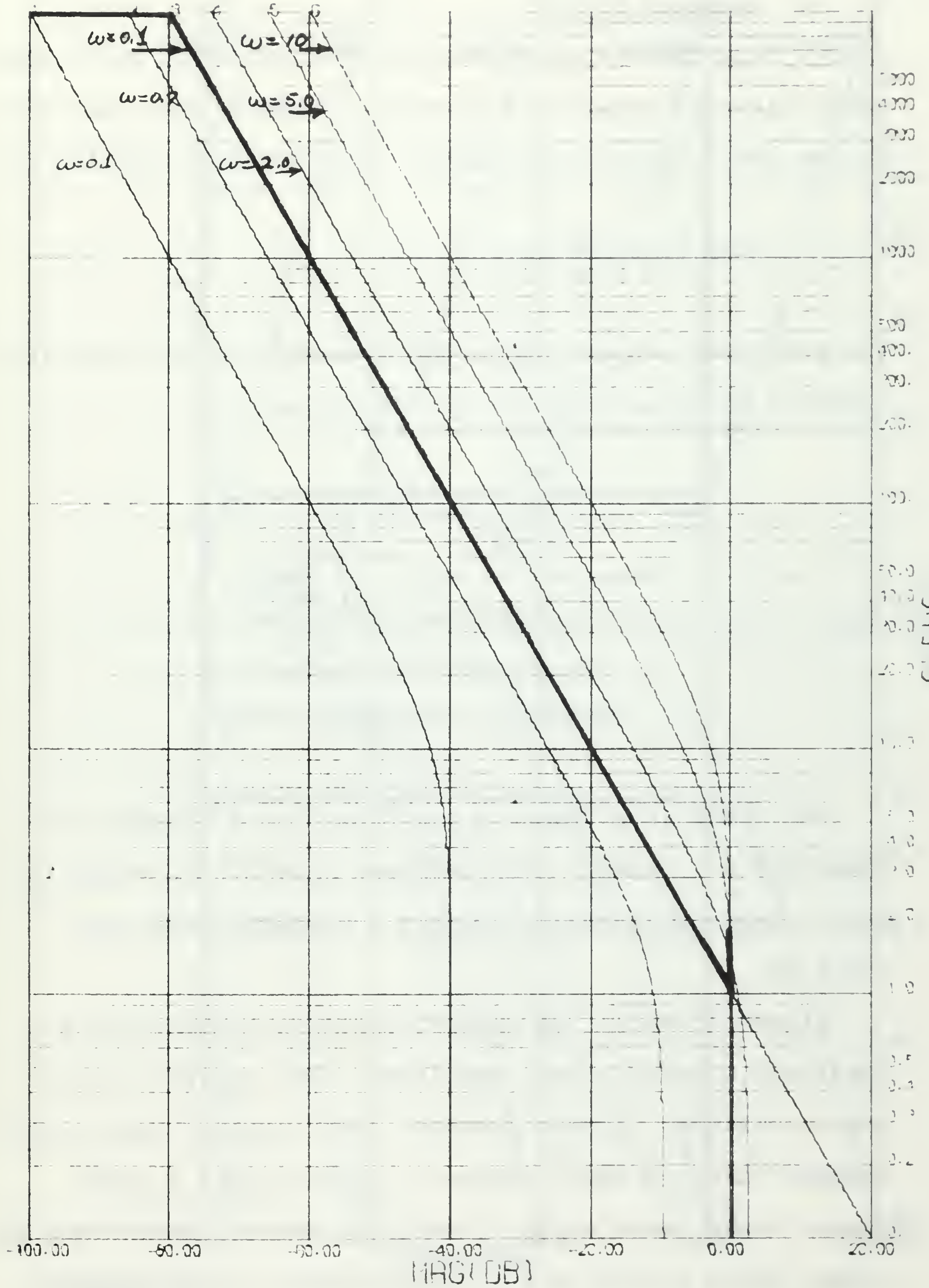


FIGURE 12.
R.A.NICHOLS,ALPHA VS MAGNITUDE(DB),PARAM=OMEGA
HIGHPASS FILTER, L=C=1.0, R=ALPHA

3. Bandpass Filter

The three-element circuit configuration for a band-pass filter is shown in Figure 13. Again L and C are set to one and R replaced by alpha. The transfer function is

$$T(s) = \frac{\alpha s}{1 + \alpha s + s^2} .$$

The Bode plot and the alpha-versus-magnitude plot are the computer plots used in the filter design.

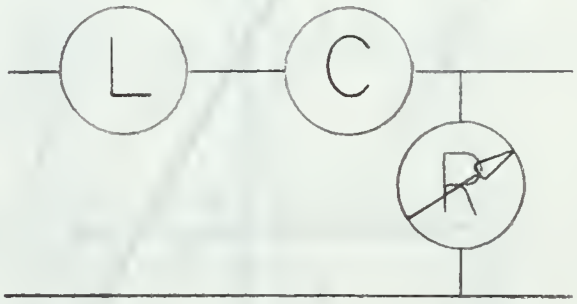


Figure 13. Bandpass Filter

The heavy line drawn on the Bode plot of Figure 14 specifies a 1.0-omega-wide bandpass symmetrical about the point where omega equals 1.0 and a stopband level of -60.0 dB.

Figure 15 shows the specifications in terms of the nonlinear element versus magnitude. The desired alpha characteristics in this case are more complex than in the lowpass case. A small change in alpha causes a large output change when alpha, itself, is small. When alpha is large, which is the case in the passband, large parmeter value changes are necessary to affect the output. This

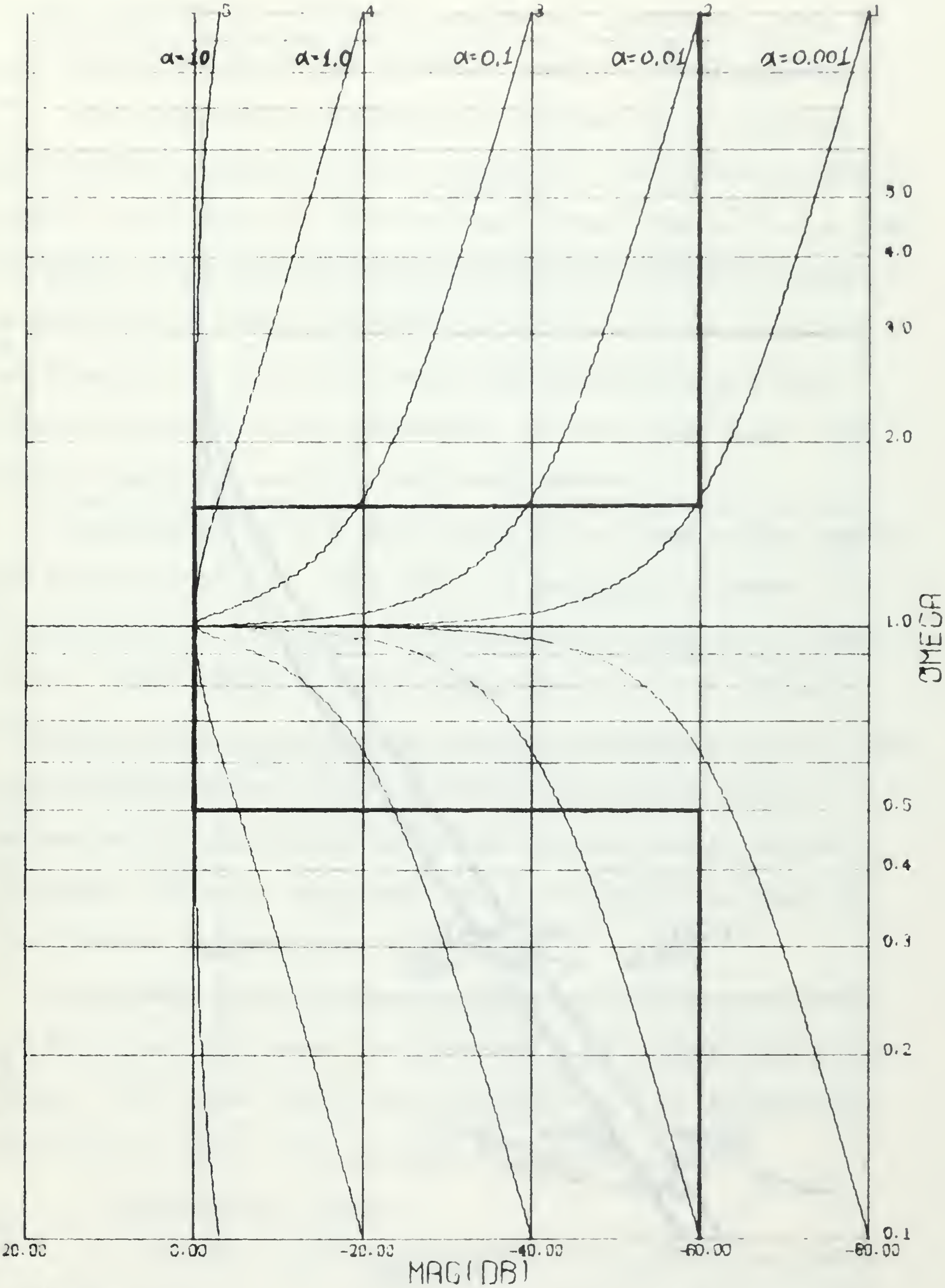


FIGURE 14.
R.A.NICHOLS, BODE PLOT, PARAMETER=ALPHA
BANDPASS FILTER, L=C=1.0, R=ALPHA

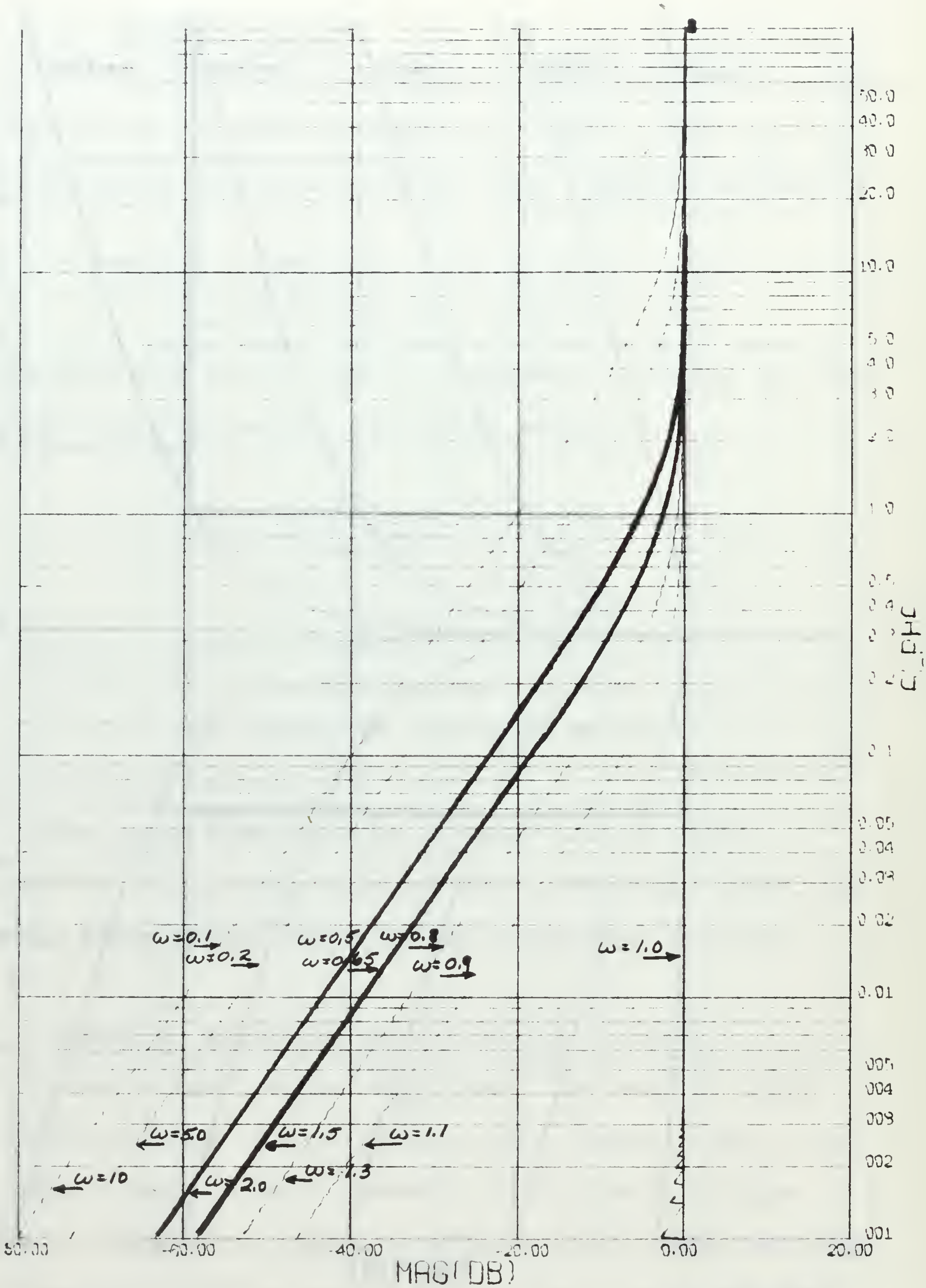


FIGURE 15.
R.A.NICHOLS, ALPHA VS MAGNITUDE(DB), PARAM=OMEGA
BANDPASS FILTER, L=C=1.0, R=ALPHA

would indicate a relatively unstable attenuation level in the stopband region but a stable level in the passband.

The width of the passband is limited by the minimum attenuation required in the stopband. The wider the passband, the higher the maximum alpha value must be for a flat response. The range of values available with the chosen element would then indicate the point where the stopband attenuation is too low to meet specifications and thus limit the width of the passband. On the other hand, the filter passband could be made very narrow.

Realization of the filter can not be done on the basis of magnitude alone. Some sort of frequency or phase sensing device is needed to determine the region of operation, either above or below omega equal to 1.0. Given a frequency sensor the circuit could be formed by letting the magnitude response level crossing initiate the alpha parameter value changes to define the end points of the passband. This is the same type of operation as used in the lowpass and highpass filters.

The phase plot is shown in Figure 16. The zero phase-shift region indicates the passband where alpha values are large. The phase below the passband would be 90 degrees leading and above the passband 90 degrees lagging.

4. <u>Band-Reject Filter</u>

A three-element band-reject filter is illustrated in Figure 17. The transfer function when L and C are set to one and R replaced by alpha is

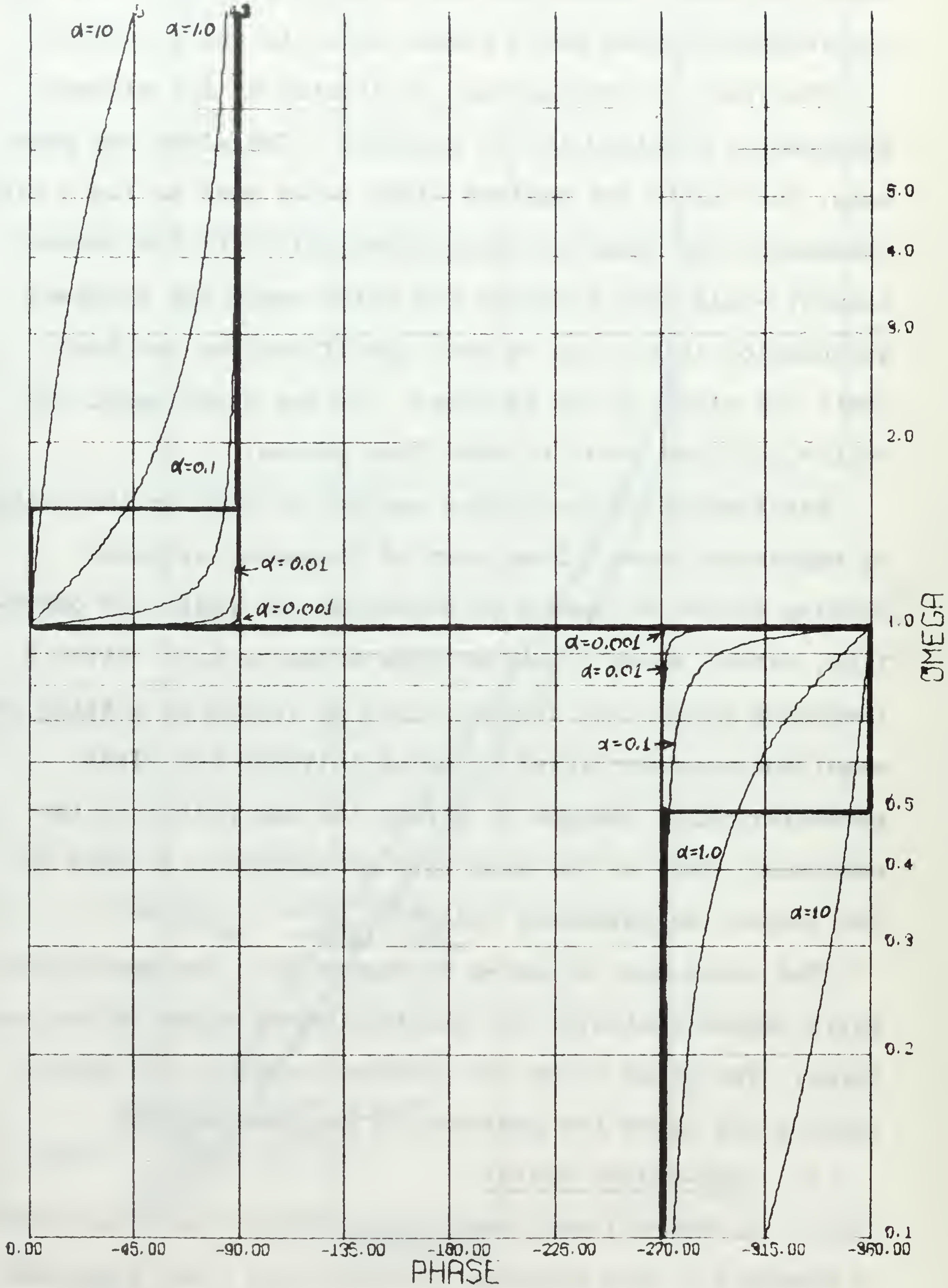


FIGURE 16.
R.A.NICHOLS, PHASE PLOT, PARAMETER=ALPHA
BANDPASS FILTER, L=C=1.0, R=ALPHA

$$T(s) = \frac{1 + s^2}{1 + s + s^2} .$$

The design of the filter calls for Bode and alpha-versus-magnitude plots.

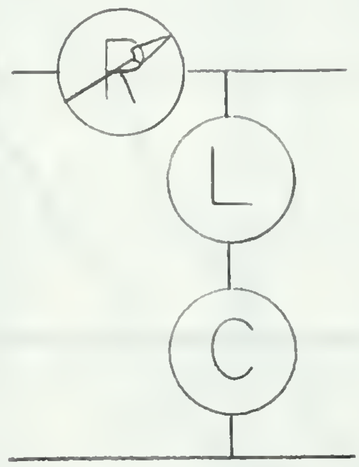


Figure 17. Band-Reject Filter

The specifications are indicated on the Bode plot, Figure 18, and the alpha-versus-magnitude plot, Figure 19. The phase plot is shown in Figure 20. The plots confirm the expected, that the band-reject filter is the inverse of the bandpass filter.

As in the case of the bandpass filter, the band-reject filter can not be realized by using magnitude response alone. It too must have a frequency or phase sensor to indicate the region of operation.

## B. FILTERS WITH MORE THAN ONE NONLINEAR ELEMENT

More complex filters may be formed using more than one nonlinear element. They may be built up by combining filter

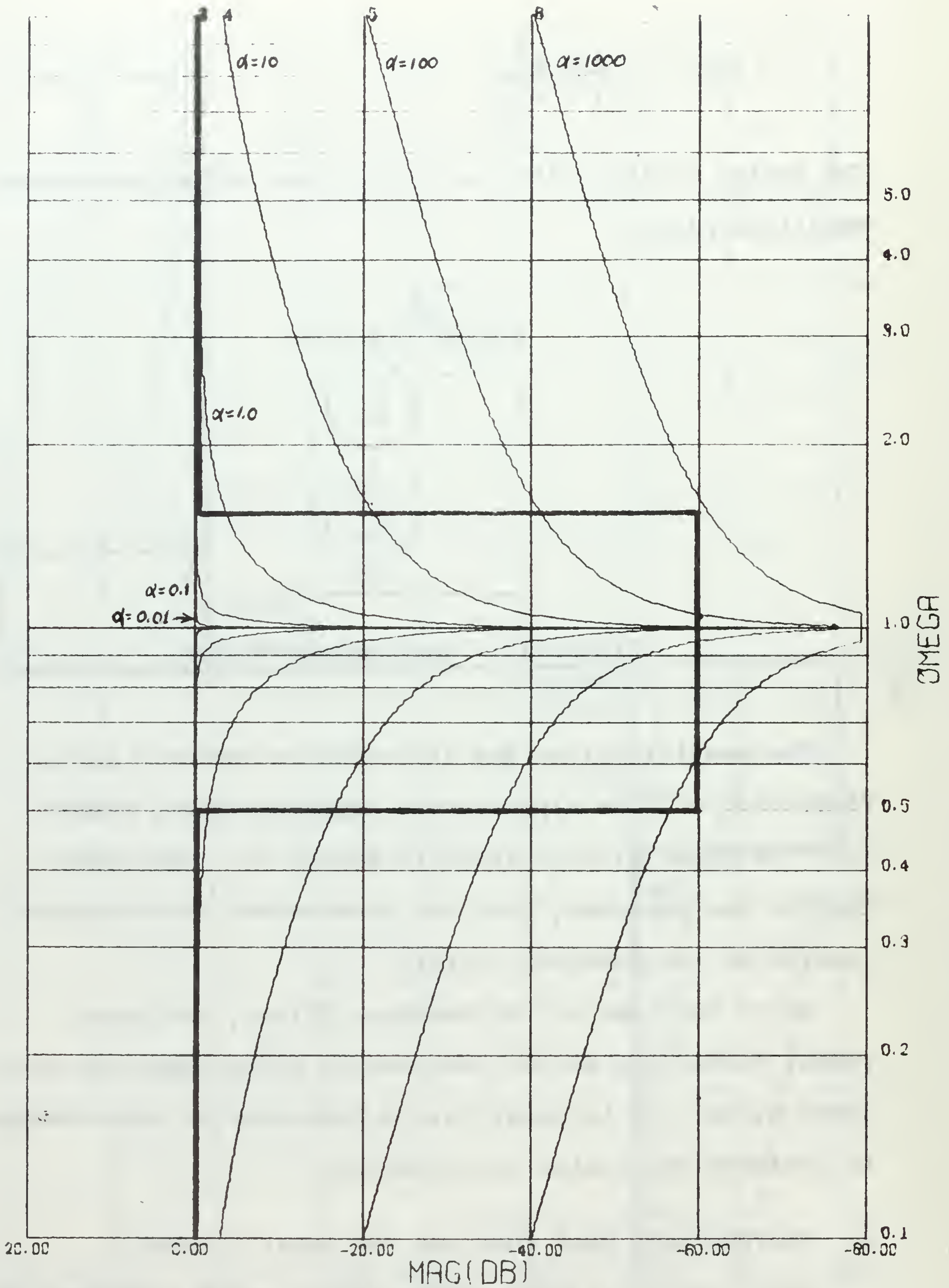


FIGURE 18.
R.A.NICHOLS, BODE PLOT, PARAMETER=ALPHA
BAND-REJECT FILTER, L=C=1.0, R=ALPHA.

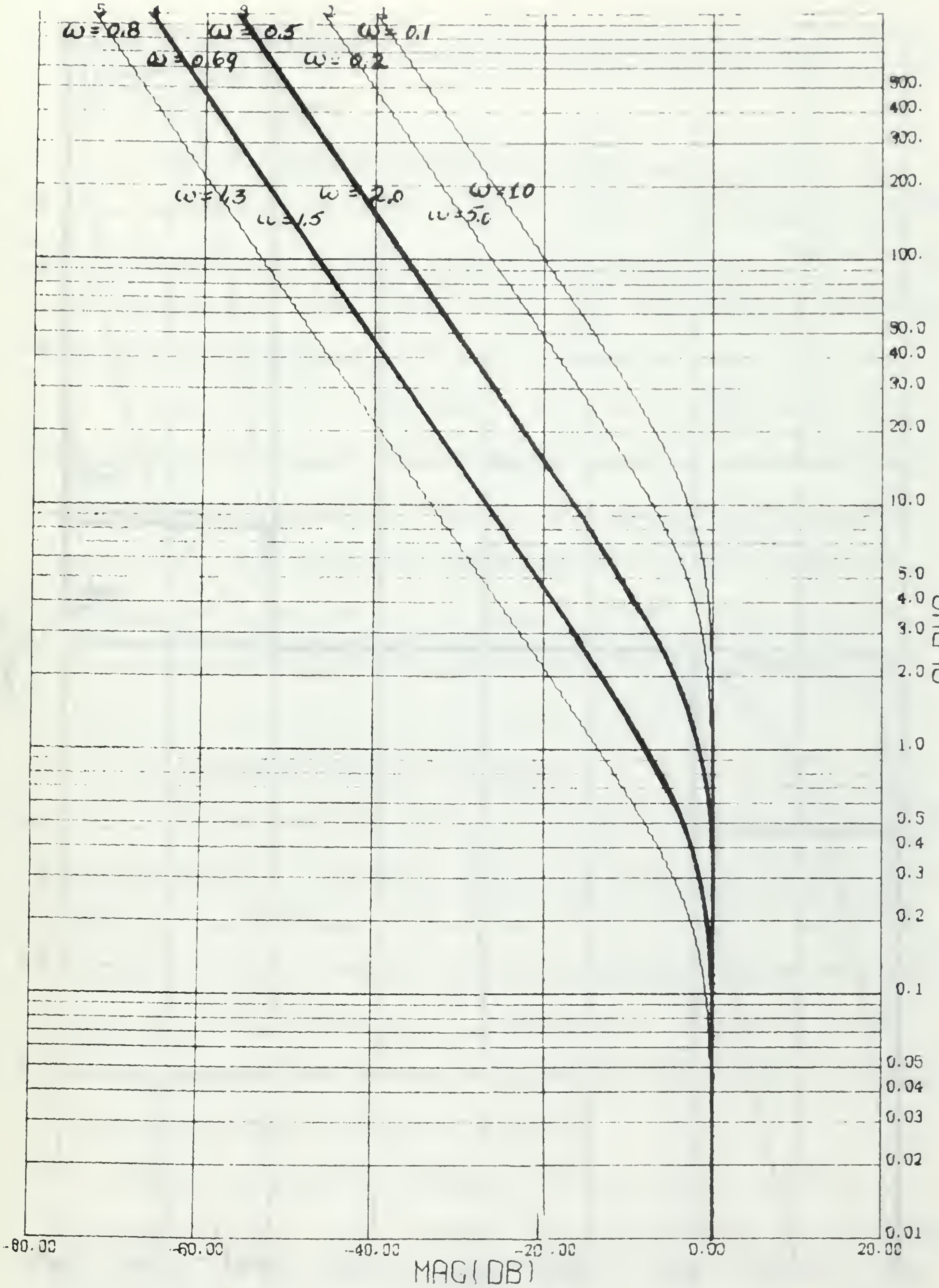


FIGURE 19.

R.A.NICHOLS.ALPHA VS MAGNITUDE(DB).PARAM=OMEGA
BAND-REJECT FILTER. L=C=1.0. R=ALPHA.

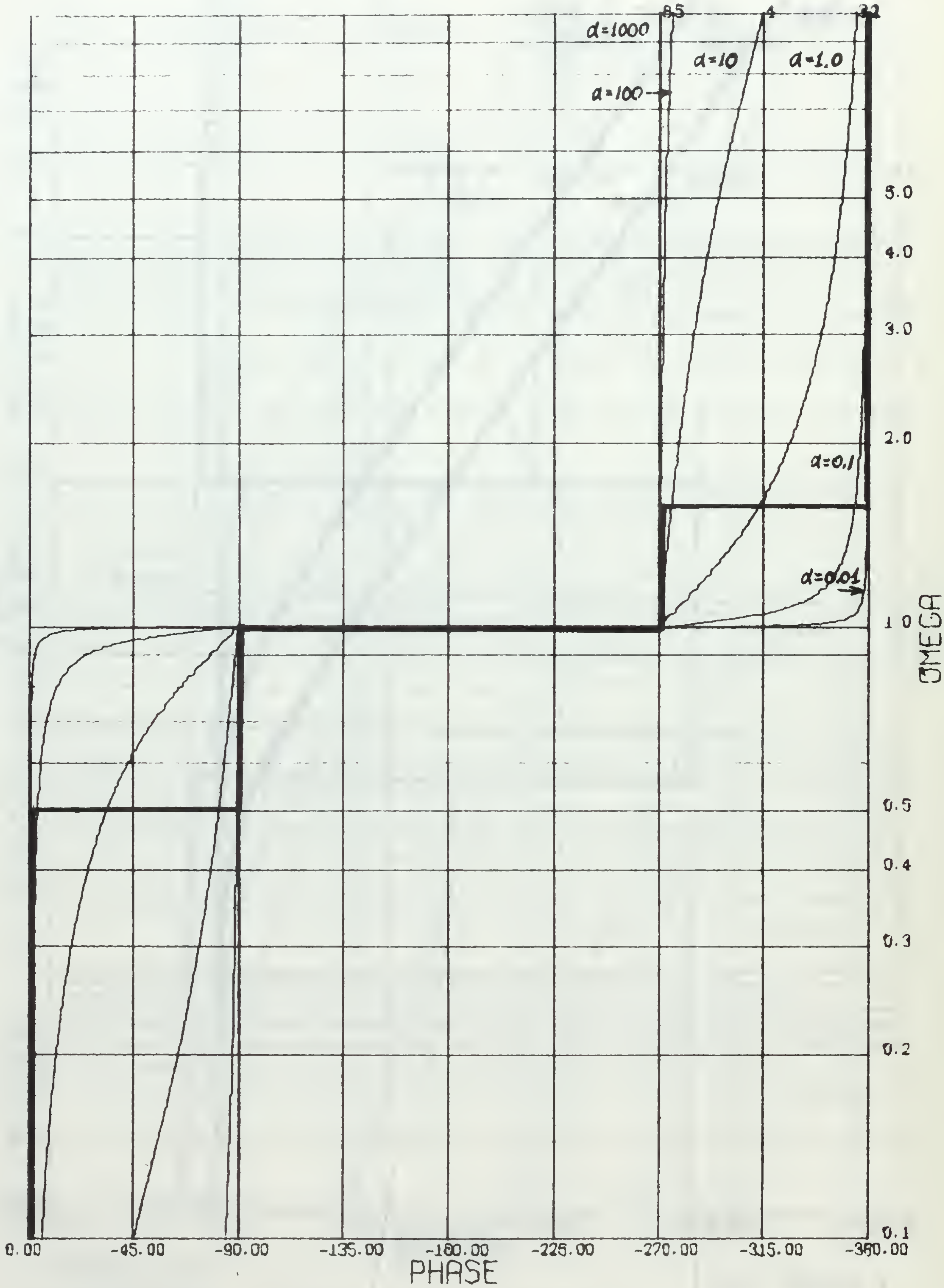


FIGURE 20.
R.A.NICHOLS, PHASE PLOT, PARAMETER=ALPHA
BAND-REJECT FILTER, L=C=1.0, R=ALPHA.

sections and/or by using several nonlinear elements in the same section.

1. Concatenating Filter Sections

Additional attenuation of undesired frequencies may be obtained by concatenating filter sections. The sections are designed as individual units and then coupled together by buffer amplifiers.

A considerable improvement over the single-section design of a bandpass filter may be made by concatenating lowpass and highpass sections. The passband end points are set in the frequency shifting step by the amount of HP-LP cutoff frequency overlapping used. A band-reject filter may be designed by inverting the bandpass filter output and adding it to the input signal.

2. Cascading Filter Sections

There are two types of cascaded filters. The first type consists of those in which the nonlinear element in each section is made a function of the transfer ratio of the total filter. This type also includes those filters in which the nonlinearity is a function of frequency since this parameter, or at least its first Fourier approximation, is common to the filter as a whole.

Figure 21 shows an example of this first type of cascaded filter. Two lowpass filter sections are cascaded with both R1 and R2 functions of the total filter output magnitude. R1 equals k times R2. Setting L, C and k to

one and replacing the R's with alpha gives a transfer function of

$$T(s) = \frac{1}{2\alpha s + (2 + \alpha^2)s^2 + 2\alpha s^3 + s^4} .$$

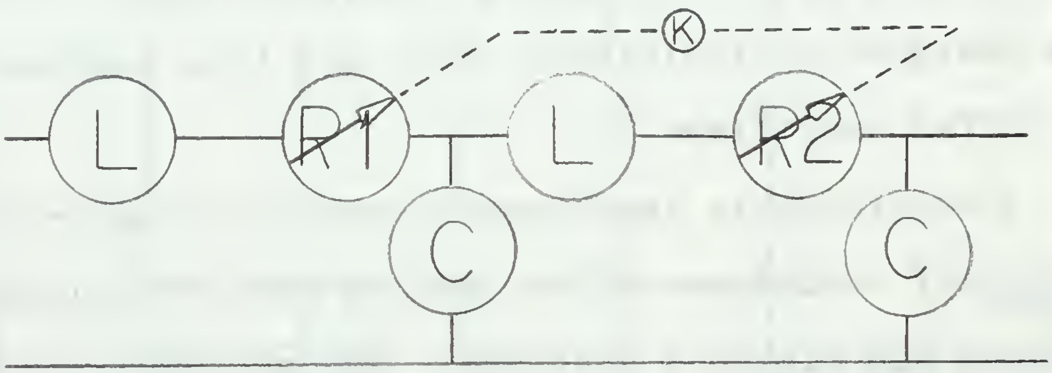


Figure 21. Cascaded Filter Sections, Type 1.

This type of cascaded filter is designed as a single filter since the transfer function contains only one parameter, alpha and it is a function of magnitude. The Bode plot for this circuit is shown in Figure 22, the alpha-versus-magnitude plot in Figure 23 and the phase plot in Figure 24. The specifications are assumed to be the same as those for the single-sections lowpass filter except that the stopband attenuation is now set to -120.0 dB.

In this filter design there is no simple approximation which would give a reasonably flat response below cutoff. The alpha value must change smoothly from 5.0 at omega equal to 0.1 to 0.1 at omega equal to 1.0. The relationship of alpha to magnitude in the omega equal to 1.0 cutoff region is shown in Figure 23. This is not a simple logarithmic

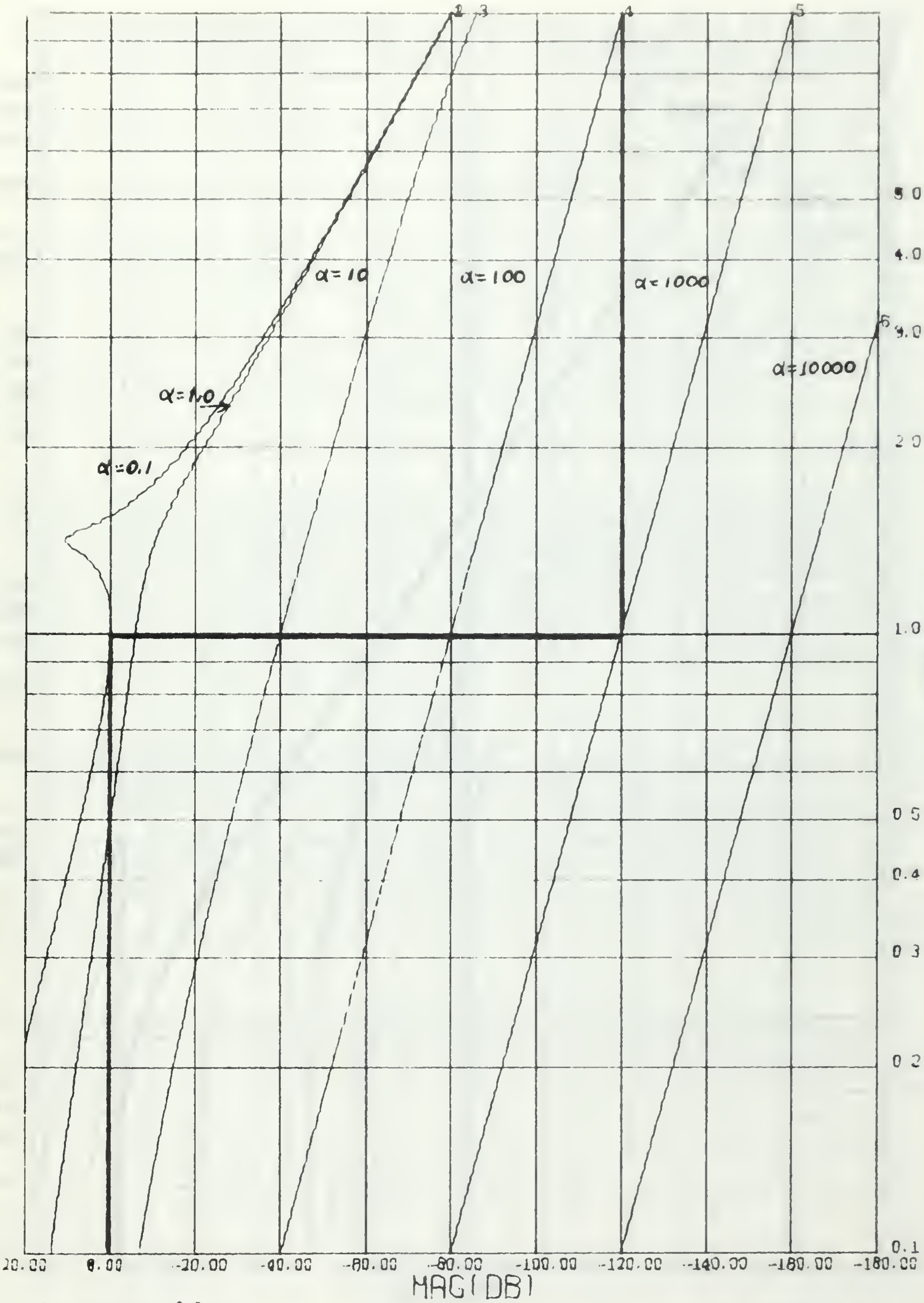


FIGURE 22.

R.A.NICHOLS. BODE PLOT. PARAMETER=ALPHA=R

2-LP SECTIONS CASCADED. TYPE 1. L=C=1.0

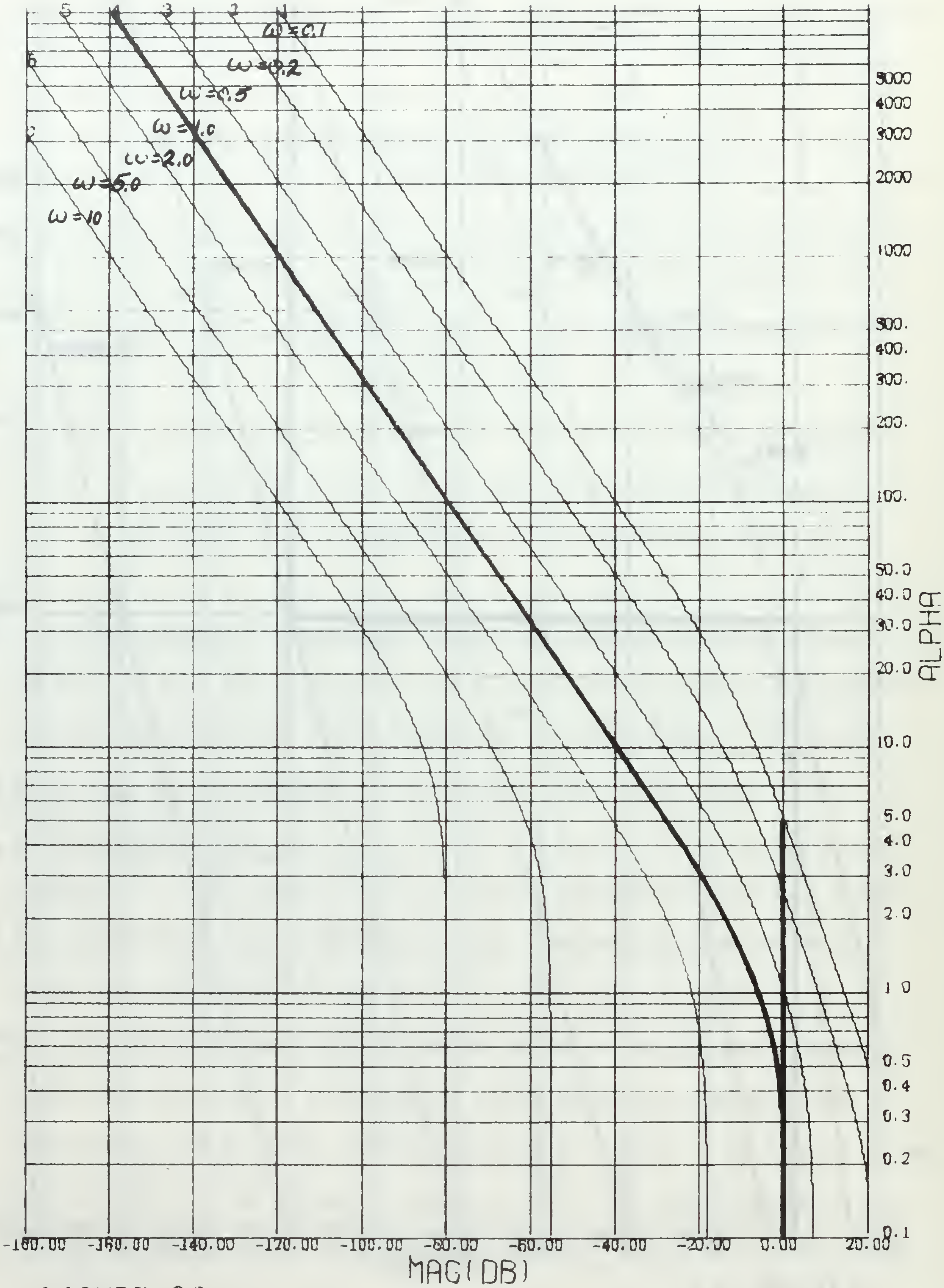


FIGURE 23.
R.A.NICHOLS,ALPHA(R) VS MAG(DB),PARAM=OMEGA
2-LP SECTIONS CASCADED, TYPE 1, L=C=1.0

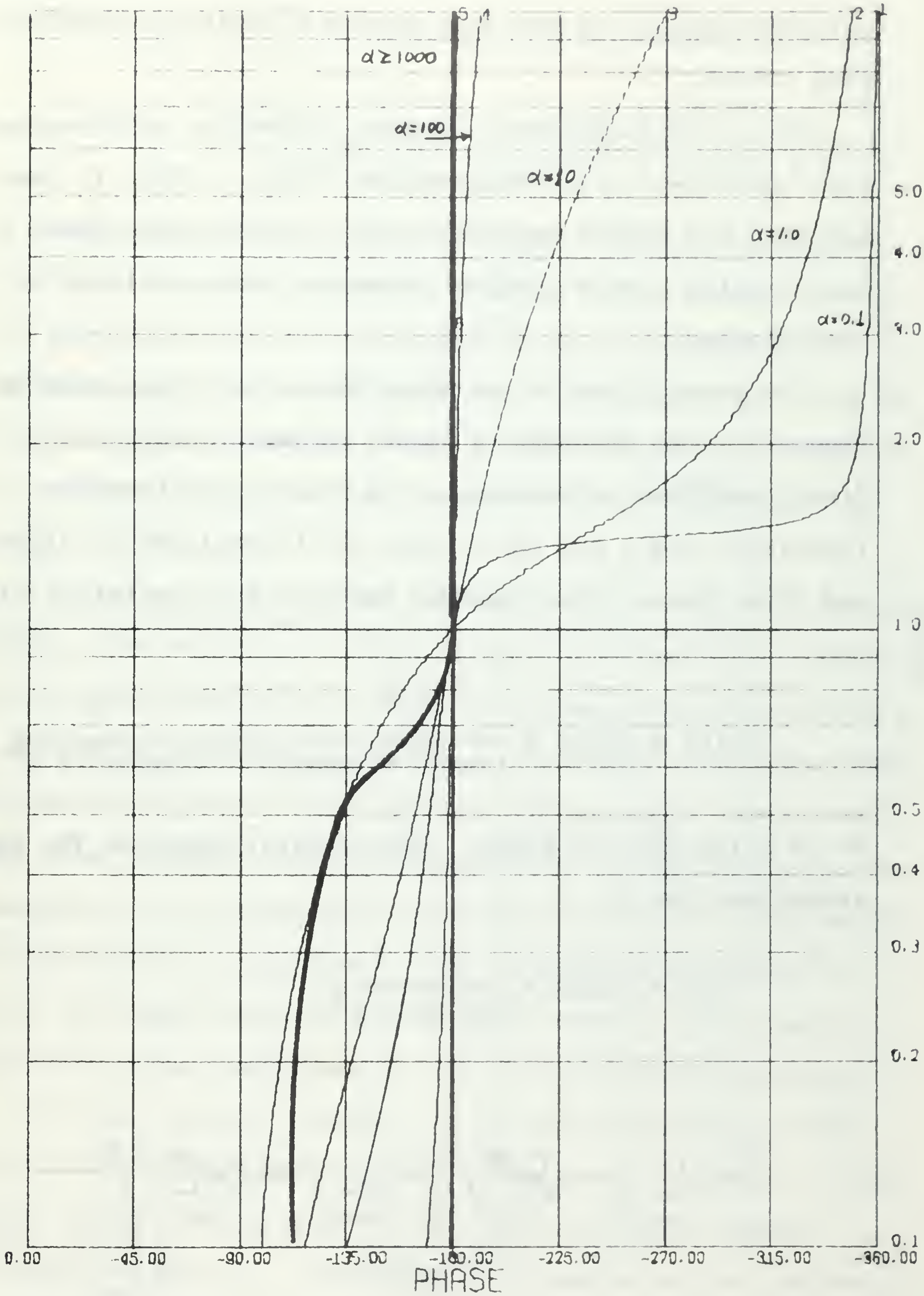


FIGURE 24.
R.A.NICHOLS, PHASE PLOT, PARAMETER=ALPHA(R)
2-LP SECTIONS CASCADED, TYPE 1, L=C=1.0

relationship as was the case with the single-section lowpass filter.

This type-1 cascaded, lowpass filter has an advantage over the two-section concatenated filter in that it does not need the buffer amplifier; but, on the other hand, it does require a more complex parameter characteristic in the passband.

The second type of cascaded filter is illustrated in Figure 25. It consists of those filters formed by the direct addition of sections. In forming the transfer function L and C are set to one, R1 is replaced by alpha and R2 by beta. The transfer function for the total filter is

$$T(s) = E3/E1 = \frac{1}{(\alpha+\beta)s + (2+\alpha\beta)s^2 + (\alpha+\beta)s^3 + s^4} .$$

R2 is a function of E3/E2. The transfer function for this second section is

$$T(s) = E3/E2 = \frac{1}{1 + \beta s + s^2} .$$

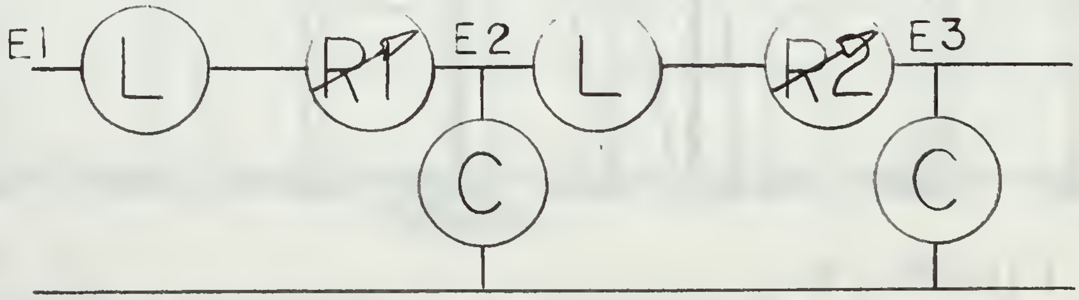


Figure 25. Cascaded Filter Sections, Type 2

In addition, R1 is a function of E2/E1 so that

$$T(s) = E2/E1 = \frac{1 + \beta s + s^2}{(\alpha+\beta)s + (2+\alpha\beta)s^2 + (\alpha+\beta)s^3 + s^4} .$$

A Bode plot for the E3/E1 transfer function is shown in Figure 26. The parameter is alpha with beta set to 1.0. The specification curve is drawn in as in the previous examples with the stopband gain level at -120.0 dB. Due to the alpha/beta symmetry in the transfer function, Figure 26 is also valid for beta as the parameter with alpha set to 1.0.

R1 is a function of an internal voltage-transfer ratio, E2/E1. The method of design in this case may be illustrated by assuming that R2 is a fixed value and that R1 is the only nonlinear element in the circuit. As before, the alpha-omega coordinate points of the desired response curve are picked off from the Bode plot and transferred to the alpha-versus-magnitude plot illustrated in Figure 27 and the required R1 characteristics determined. The only difference here is that the magnitude used is the E2/E1 voltage ratio rather than the output magnitude of the E3/E1 transfer function.

In this filter, however, R2 is also nonlinear so the plot of R2 versus E3/E2 magnitude must be considered. This plot is the same as that of the single-section lowpass filter in Figure 5. Therefore, realization of the passband region may be simplified by setting R2 at 1.0 and causing R1 to follow the characteristic curve in Figure 27. The values

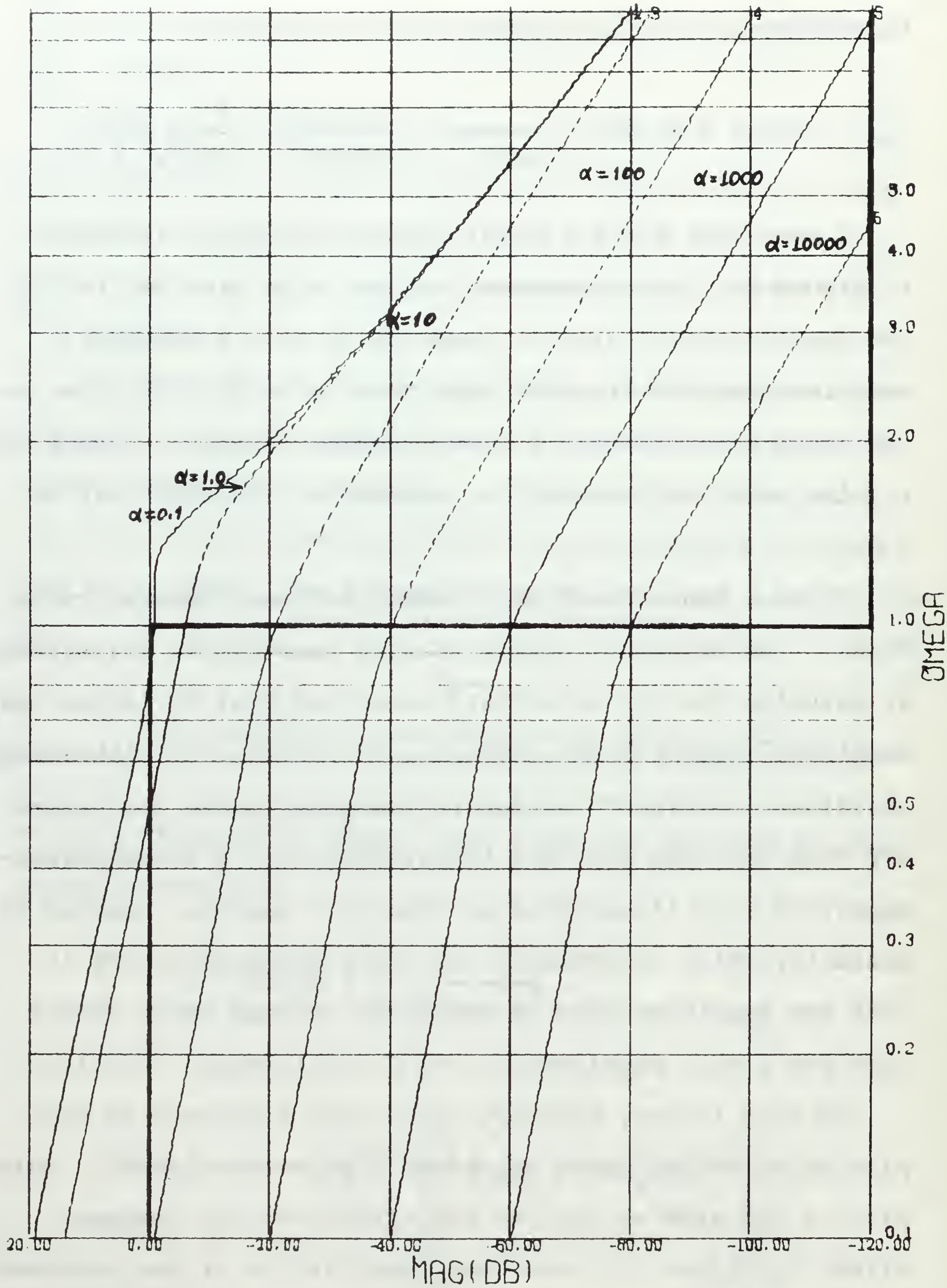


FIGURE 26.
R.A.NICHOLS, BODE PLOT, PARAMETER=ALPHA=R1
(E3/E1),2-LP SECTIONS CASCADED.R2=BETA=1.0=L=C.

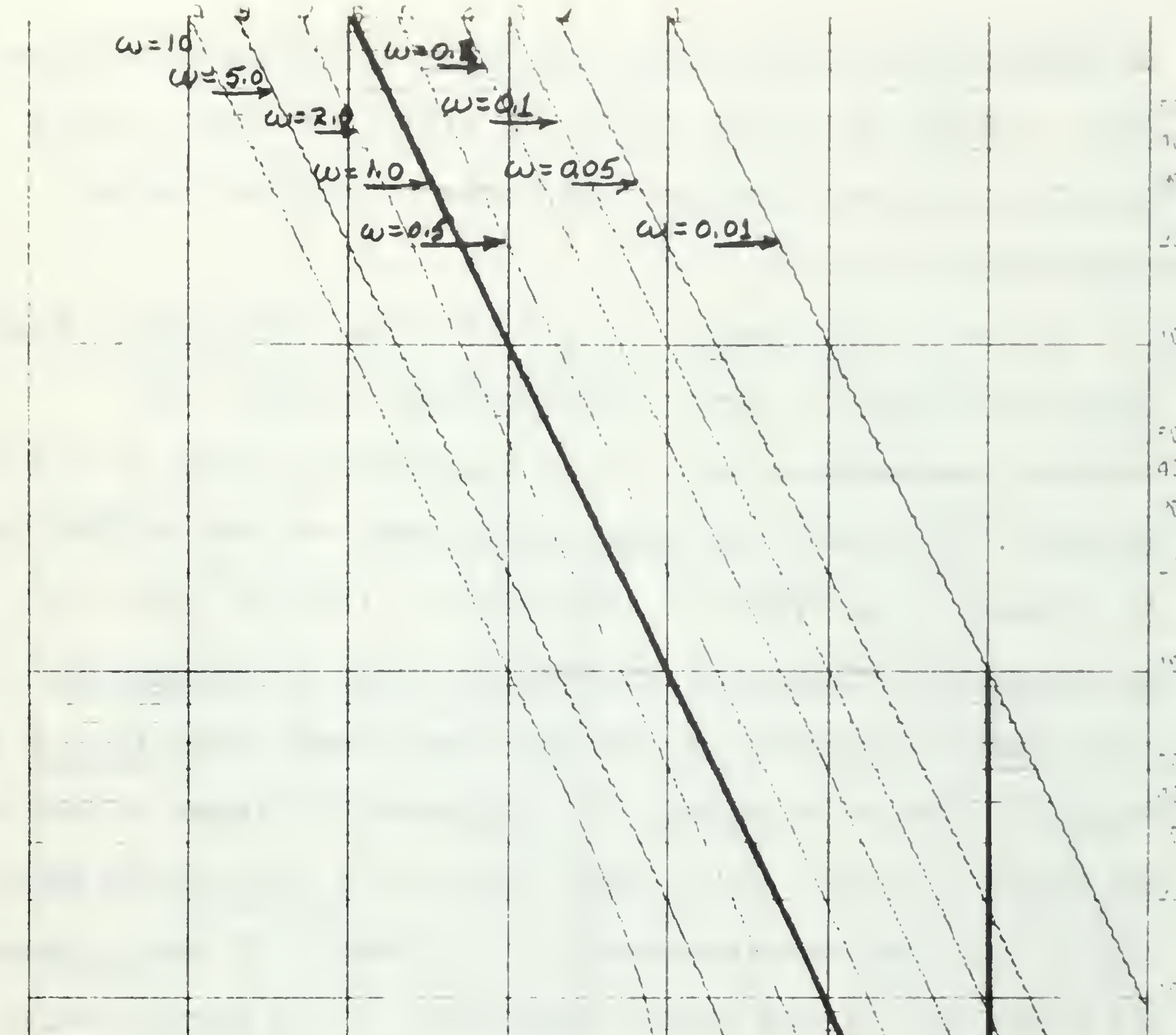


FIGURE 27.
R.H. NICHOLS, ALPHA(R1) VS MAG(DB), PARAM=OMEGA
(E2/E1), 2 LP SECTIONS CASCADED, R2-L=C=1.0

of alpha to be used in the passband region are determined from Figure 28. This figure is a plot of alpha versus beta for passband frequencies where the filter output magnitude is 0.0 dB.

In the cutoff region R1 and R2 change from their passband low values to their stopband high values. R2 follows the omega equal to 1.0 logarithmic curve to the minus 80.0-dB level at beta equal to 10,000. A four-decade range R1 element goes from 0.1 at 0.0 dB to 1000 at -60.0 dB. In so doing it follows the hand-drawn curve in Figure 29. This curve is a composite of the line for omega equal to 1.0 in Figure 27 where R2 equals 1.0 and that of Figure 29 where R2 equals 10,000, The chosen values are determined from the E2/E1 alpha-versus-beta plot of Figure 30 where omega is fixed at 1.0 and output magnitude is the curve family parameter.

In the stopband region R1 may be left at 1000 and R2 at 10,000. The resultant attenuation at cutoff is -140.0 dB from which it falls off at 40 dB/decade. Figure 31 is the Bode plot of E3/E1 for the stopband region above omega equal to 1.0.

Figure 32 illustrates the phase response for passband values of alpha and beta while Figure 33 shows the phase response for stopband values.

Figure 34 and 35 show what the output magnitude and phase response would have been if two lowpass circuits had been added together directly. The response is not the

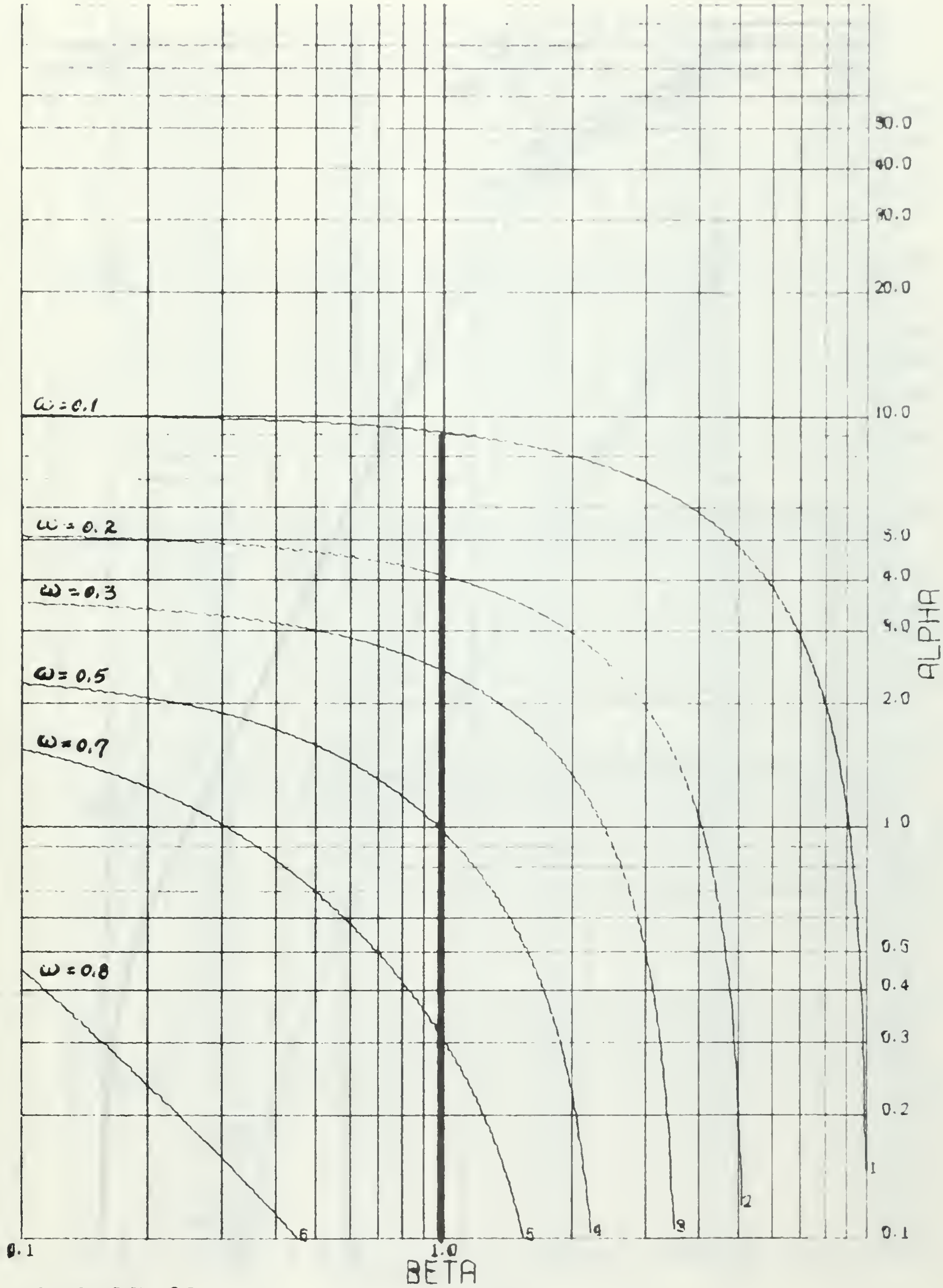


FIGURE 28.
R.A.NICHOLS,ALPHA(R1) VS BETA(R2),PARAM=OMEGA
2-LP SECT CASCADED.L=C=1.0.MAG(DB)=0.0

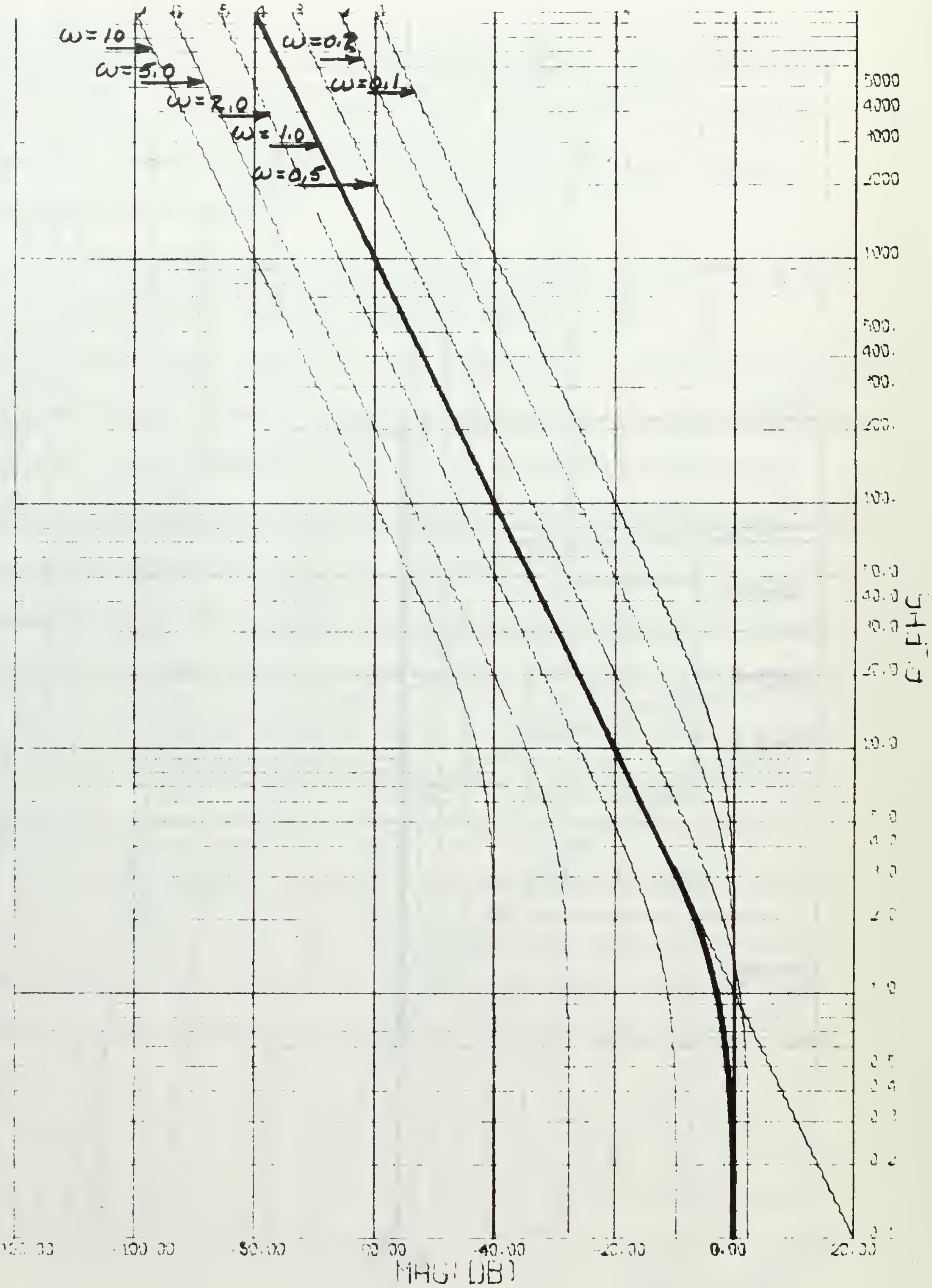


FIGURE 29.
R.A.NICHOLS,ALPHA(R1) VS MAG(DB),PARAM=OMEGA
(E2 E1),2-LP SECT CASCADED,L=C=1.0,R2=10000.0

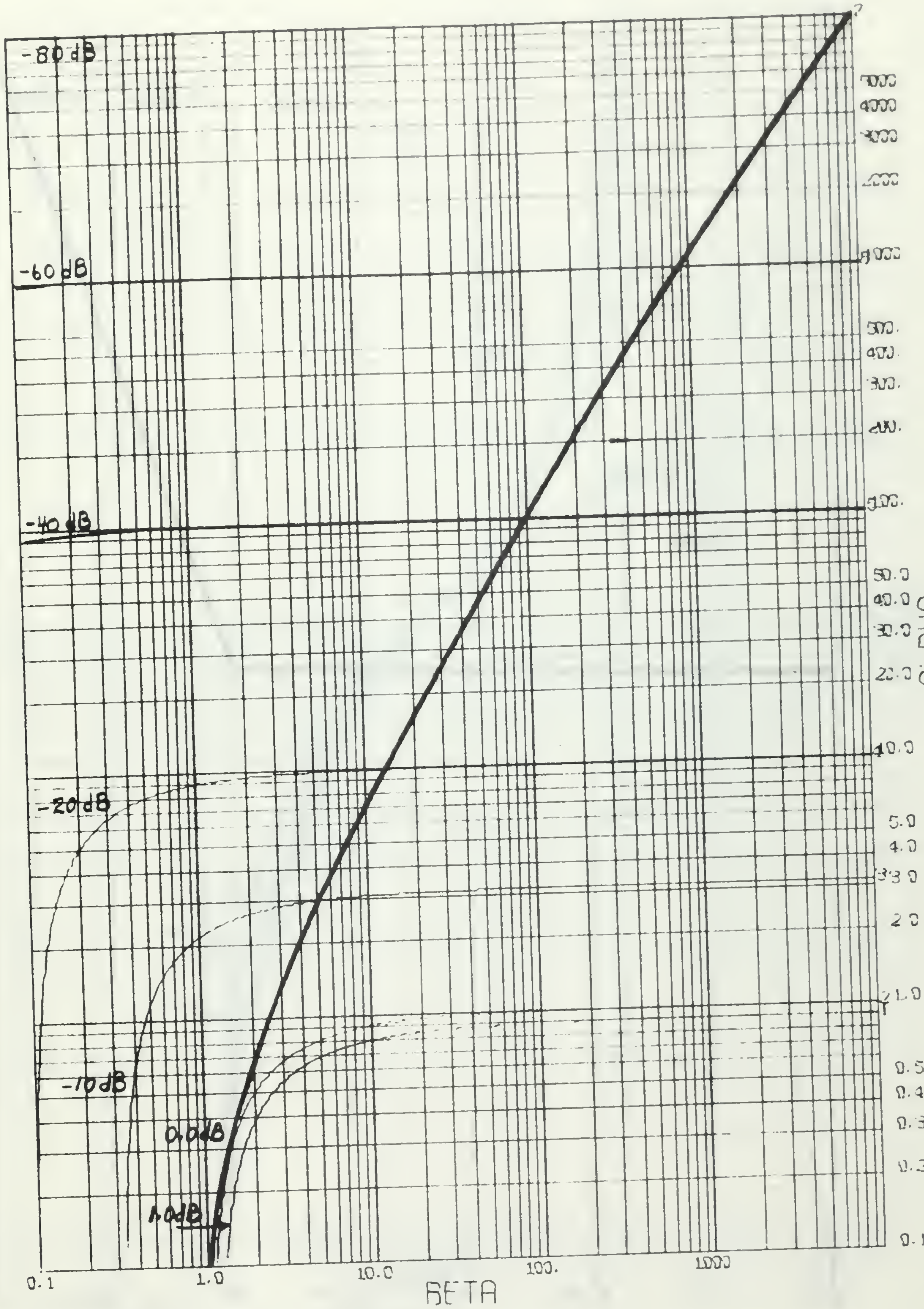


FIGURE 30.
R. A. NICHOLS, ALPHA(R1) VS BETA(R2), PARAM=MAG(DB)
(E2/E1), 2-LP SECT CASCADED, L=C=1.0, OMEGA=1.0.

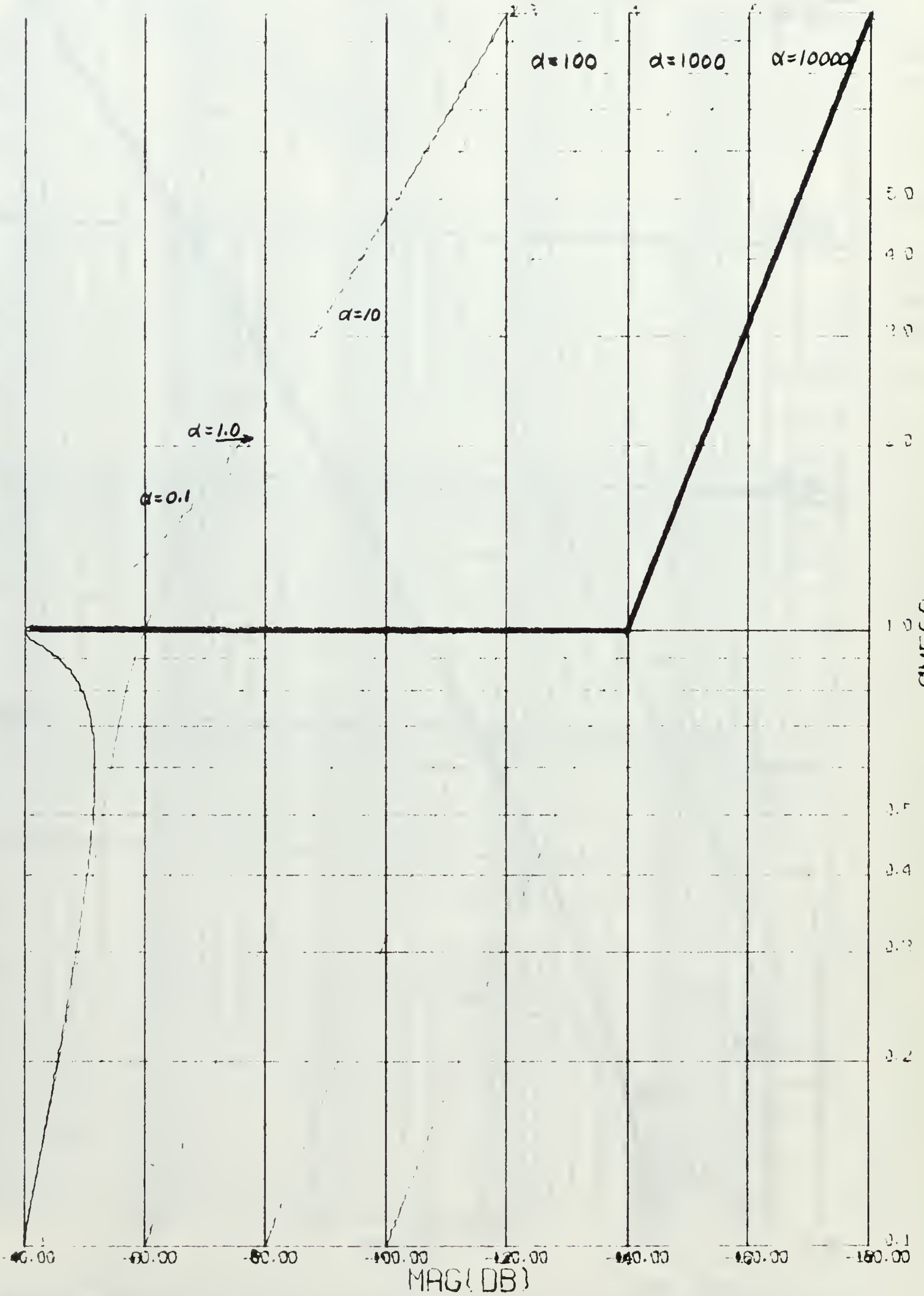


FIGURE 31.
R.A.NICHOLS, BODE PLOT, PARAMETER=BETA=R2
(E3-E1),2-LP SECT CASCADED,R1=1000.0,L-C=1.0

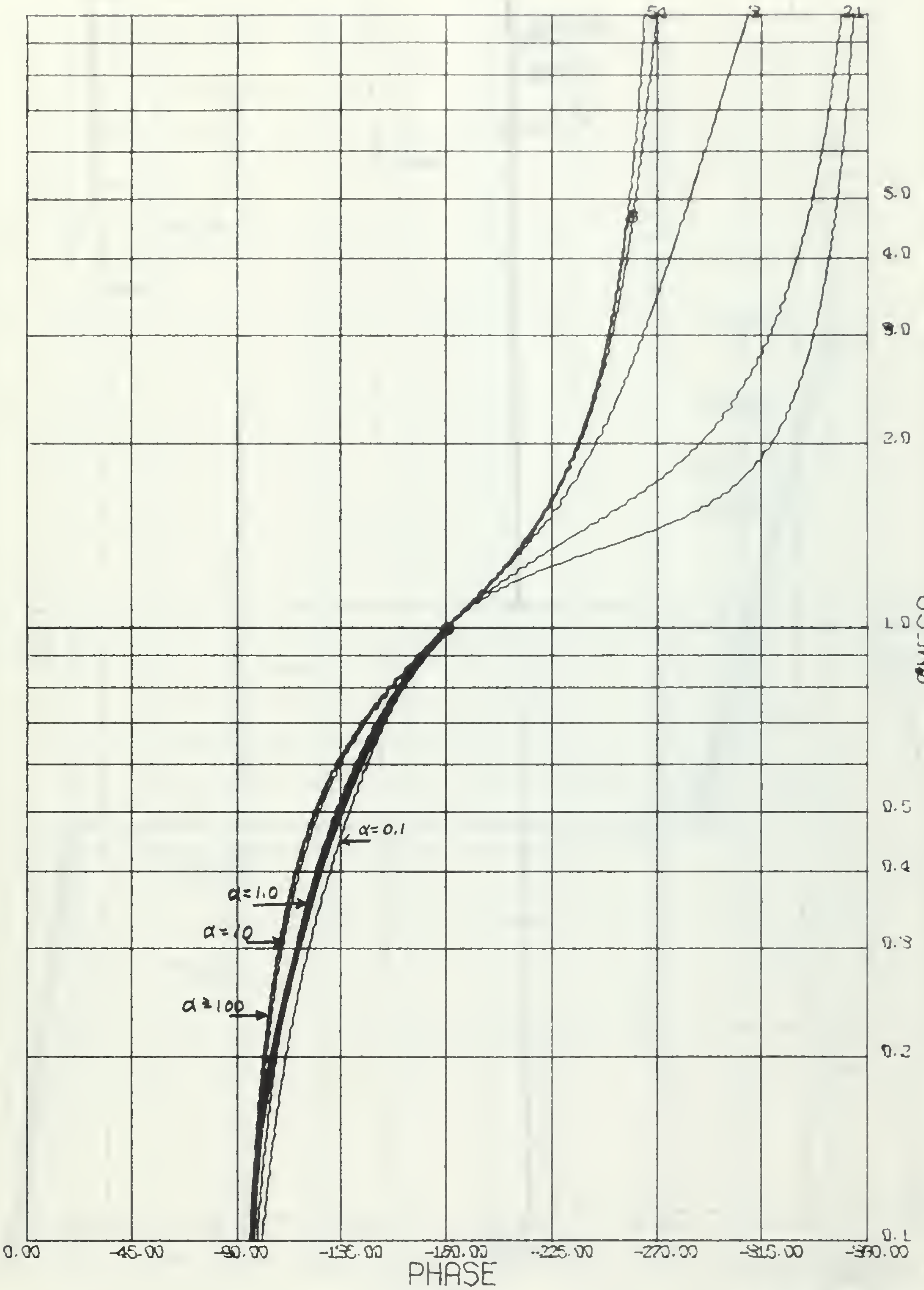


FIGURE 32.
R.A.NICHOLS. PHASE PLOT, PARAMETER=BETA=R2
(E3/E1),2-LP SECT CASCADED,R1=ALPHA=1.0=L=C

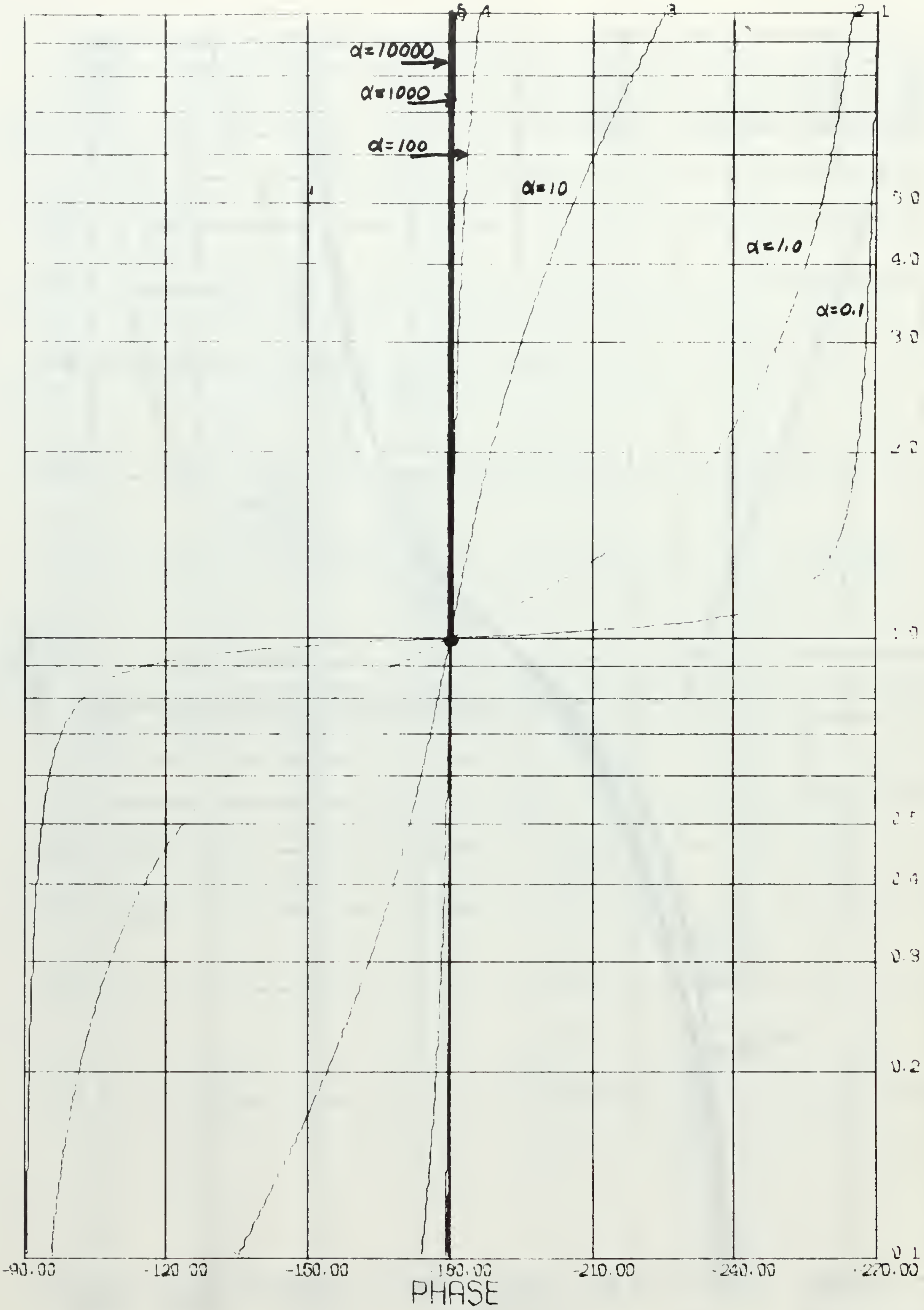


FIGURE 33.
R.A.NICHOLS, PHASE PLOT, PARAMETER=BETA=R2
(E3/E1), 2-LP SECT CASCADED, R1=1000.0, L=C=1.0

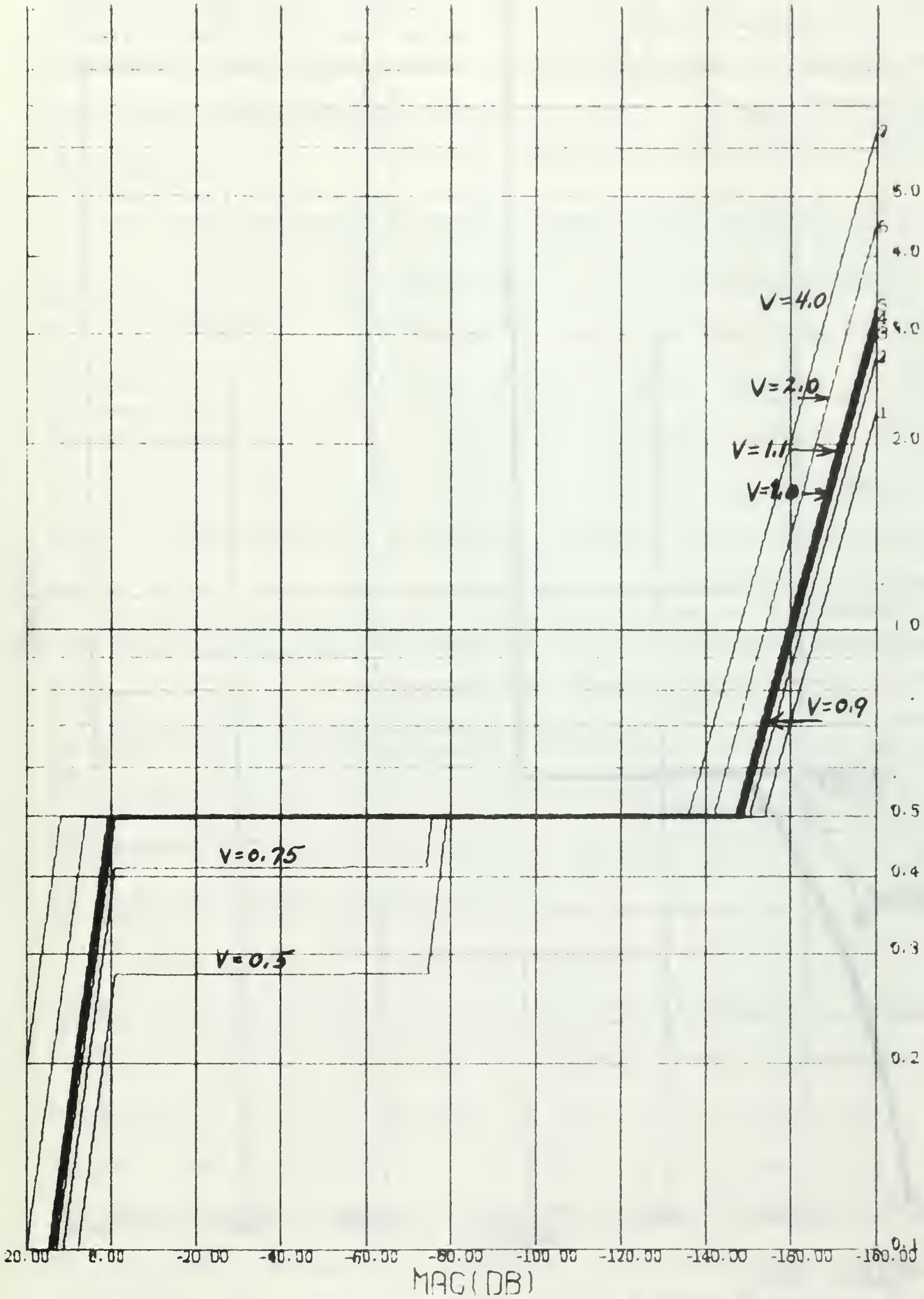


FIGURE 34.
R.A.NICHOLS.BODE PLOT.PARAM=(VINPUT*1.0)
2-LP SECT CASCADED.TYPE 2.SIMULATION CHECK

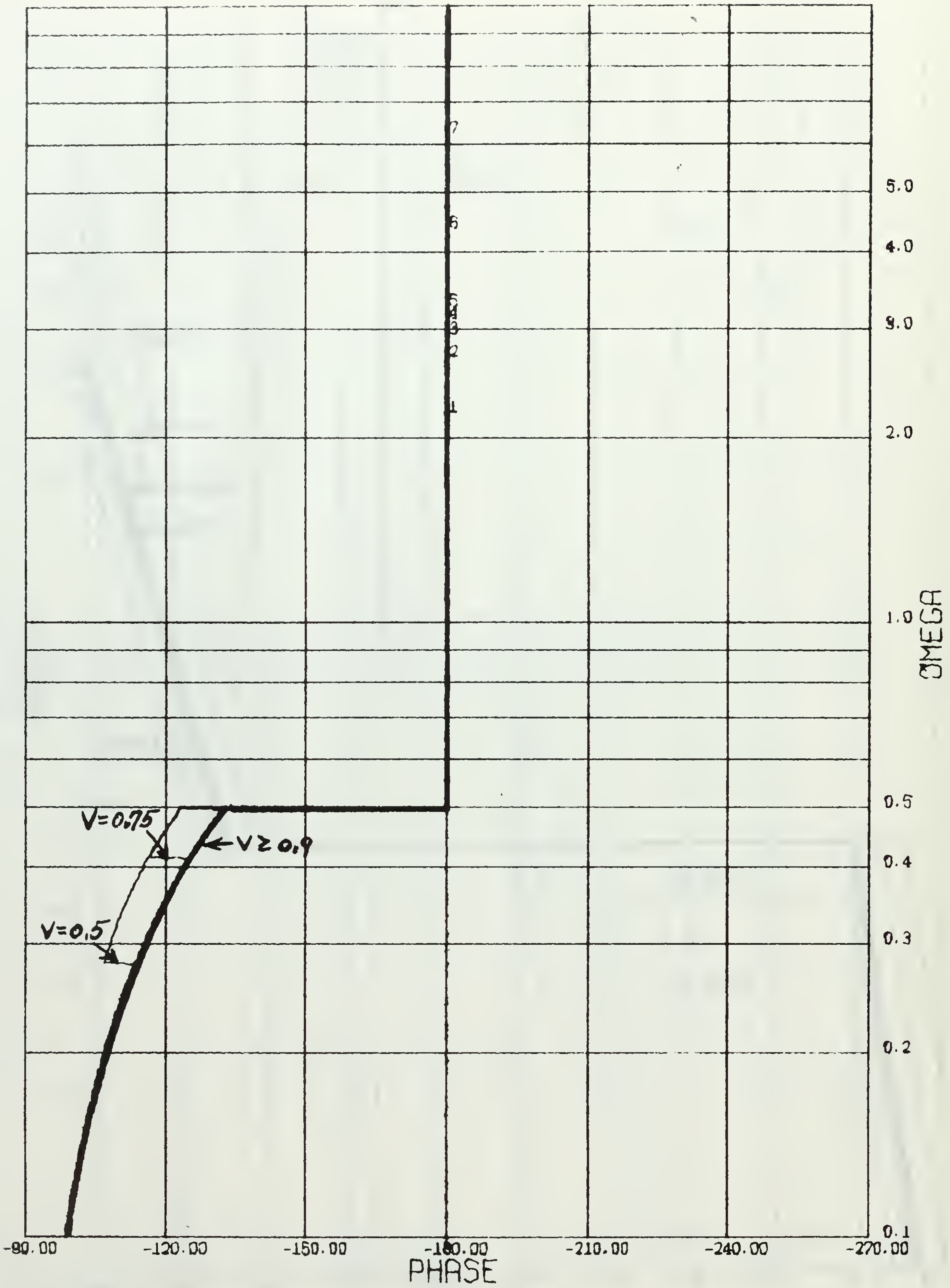


FIGURE 35.
R.A.NICHOLS,PHASE PLOT,PARAM=(VINPUT*1.0)
2-LP SECT CASCADED,TYPE 2,SIMULATION CHECK

same as would have been obtained by concatenating the two sections. The concatenated filter is easier to realize than a type-two cascaded filter and has a greater stopband attenuation.

3. Two Nonlinear Elements in the Same Section

A third configuration for two nonlinear elements may be obtained by letting both the R and the C in the basic lowpass filter be nonlinear elements and functions of the filter transfer ratio. The design procedure is the same as that of the previous sections. Bode plots are made for each of the nonlinear elements with the other set to a best guess value. From the specifications and the Bode plot information minimum and maximum values for the elements are chosen to use in a design trial. The information is transferred to parameter-versus-magnitude plots and the nonlinear element characteristics determined. The interdependence of R and C requires a composite curve to be used in defining these characteristics.

Figure 36 is the alpha-versus-beta plot for omega equal to 1.0. Alpha represents the nonlinear R which is assumed to have a four-decade range of values. Beta represents the nonlinear C which is assumed to have a two-decade value range. The specifications are those of a lowpass filter. The heavy line in Figure 36 indicates the chosen alpha/beta relationship in the filter cutoff region. Figures 37 and 38 are the alpha- and beta-versus-magnitude plots with the element characteristic curves drawn in. Computer simulation

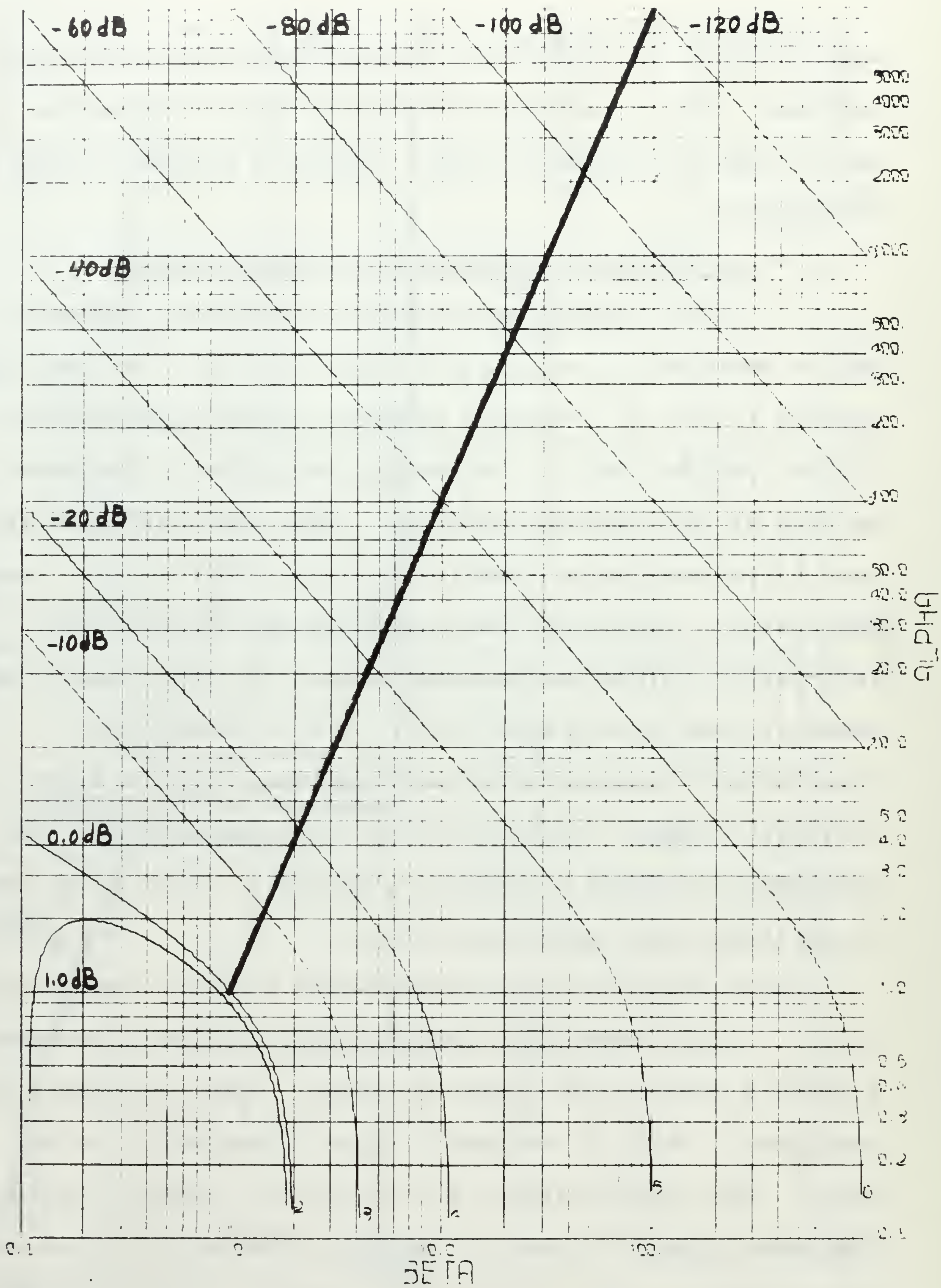

FIGURE 36.
R.A.NICHOLS.ALPHA VS BETA.PARAMETER=MAG(DB)
LP FILTER.L=1.0.OMEGA=1.0.R=ALPHA.C=BETA

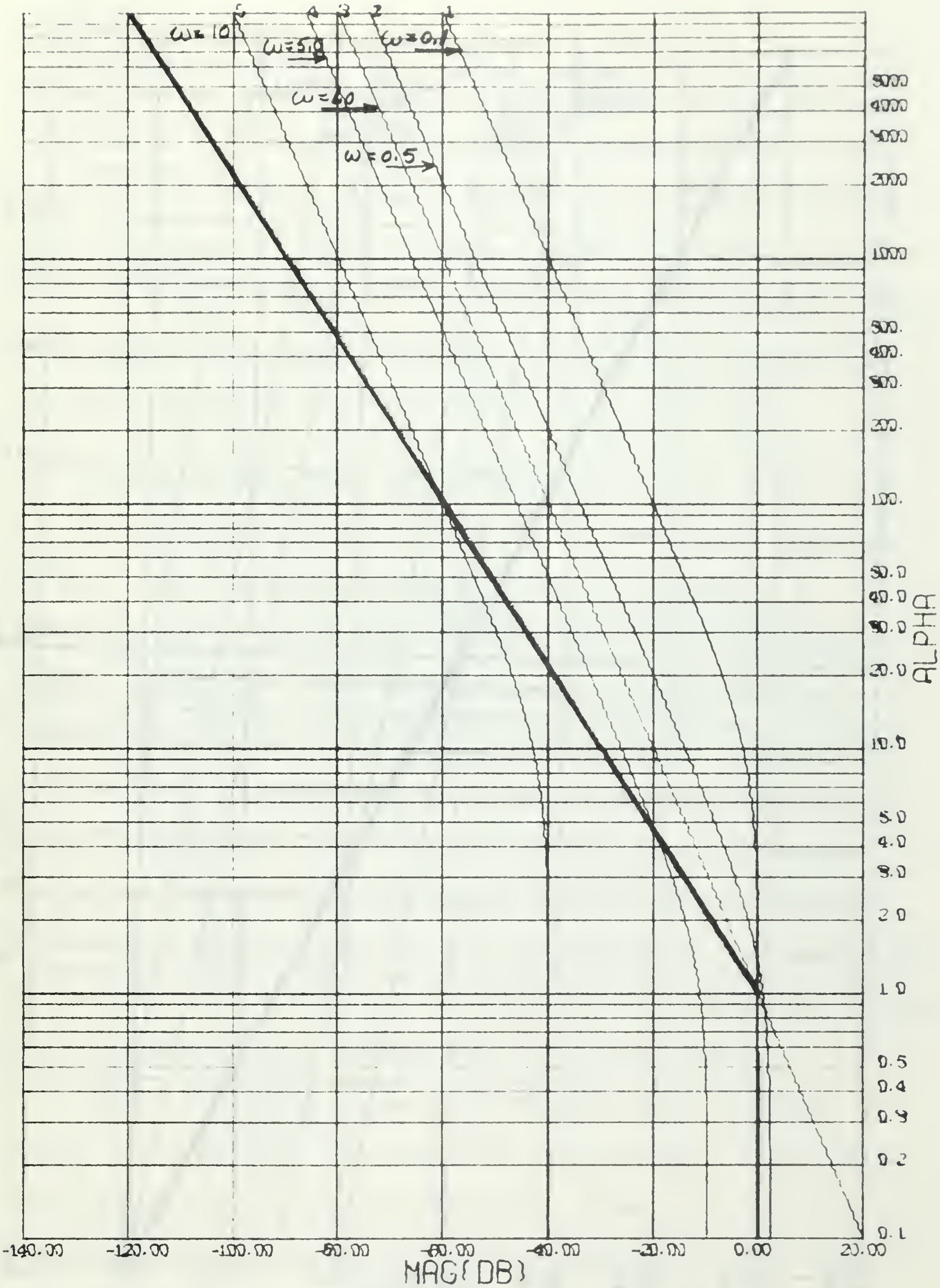


FIGURE 37.
R.A.NICHOLS, ALPHA VS MAG(DB), PARAM=OMEGA
LOWPASS FILTER, L=1.0, R=ALPHA, C=BETA=1.0

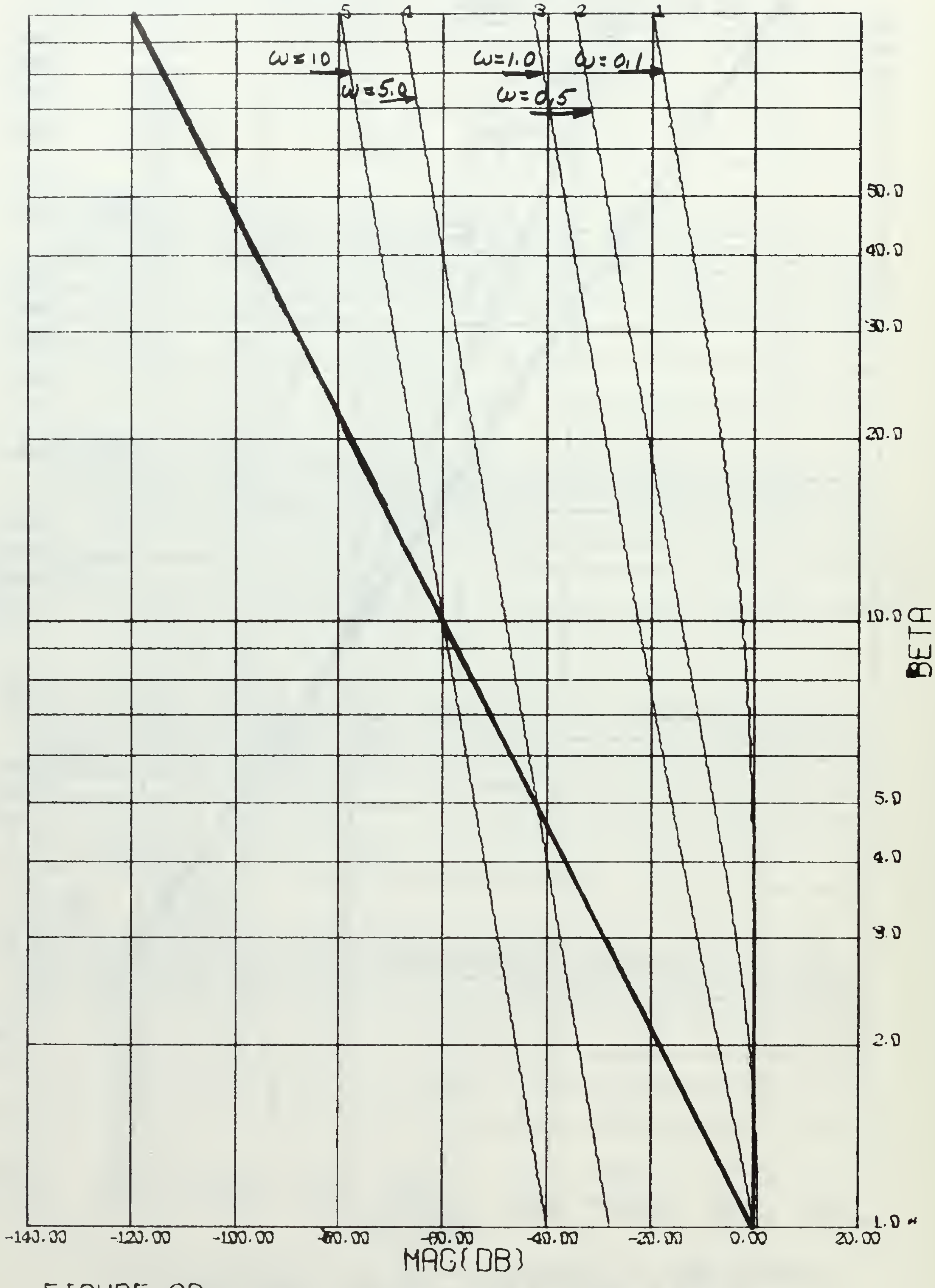


FIGURE 38.
R.A.NICHOLS.BETA VS MAG(DB).PARAMETER=OMEGA
LOWPASS FILTER.L=1.0,R=ALPHA=1.0.C=BETA

results for this design are shown in Figure 39 for output magnitude and in Figure 40 for the phase. Alpha and beta are set to 1.0 when the output magnitude is greater than -1.0 dB and allowed to go to 10,000 and 100, respectively, when the magnitude drops below -1.0 dB. As before, the -1.0 dB level is used to represent a lag in changeover for the change initiation at 0.0 dB. The cutoff attenuation is 120.0 dB while the phase response is nonlinear.

## C. DESIGNING FOR PHASE RESPONSE

When the filter specifications concern phase response rather than output magnitude the desired response is drawn in on the phase plot rather than on the magnitude-versus-omega plot. The alpha-omega points are still transferred to the alpha-versus-magnitude plot for definition of the characteristics of the nonlinear element when it is a function of the filter output transfer function. Where the magnitude of the output is of interest the same alpha-omega points may also be transferred to the magnitude-versus-omega plot for an indication of the magnitude response corresponding to the filter phase specifications.

Figure 41 is the phase plot for the basic lowpass filter of Figure 3. The heavy line represents a specification calling for a linear 180-degree phase response between omega equal to 0.2 and 1.8. The alpha-versus-magnitude plot is shown in Figure 42. The magnitude characteristic

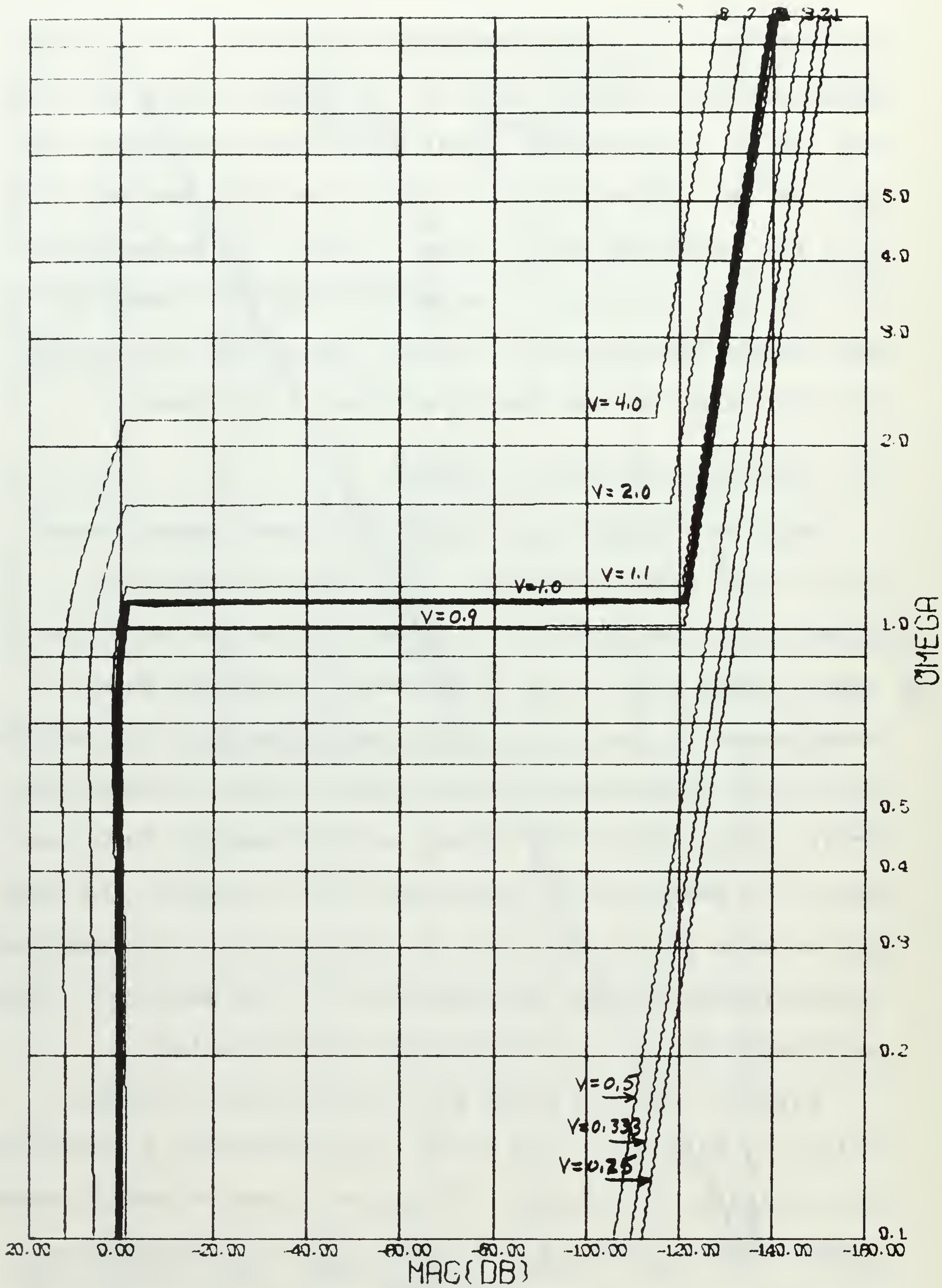


FIGURE 39.
R.A.NICHOLS,BODE PLOT,PARAM=(VINPUT*1.0)
LP FILTER SIMULATION CHECK,L=1.0,R=ALPHA,C=BETA

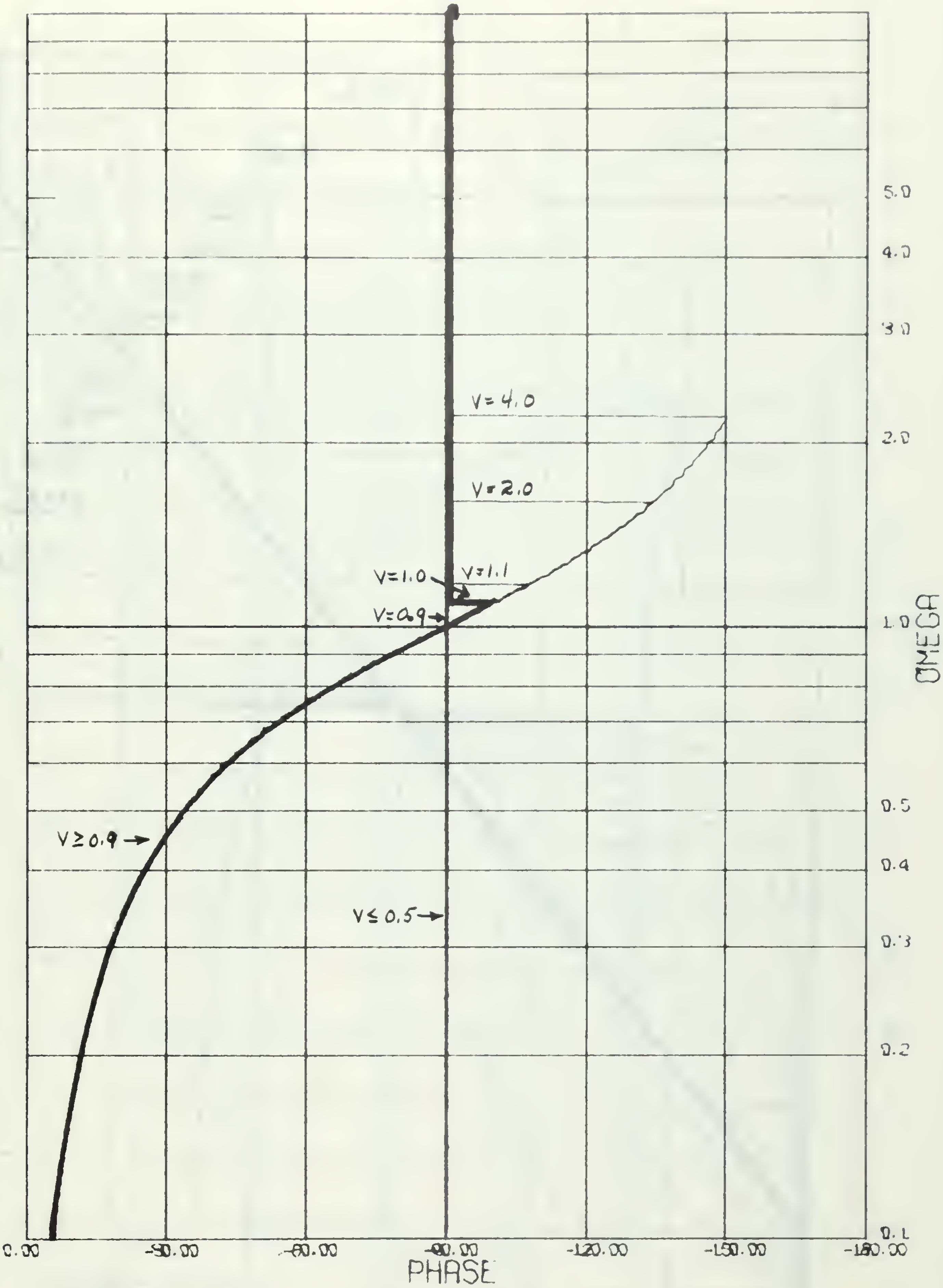


FIGURE 40.
R.A.NICHOLS,PHASE PLOT,PARAM=(VINPUT*1.0)
LP FILTER SIMULATION CHECK,L=1.0,R=ALPHA,C=BETA

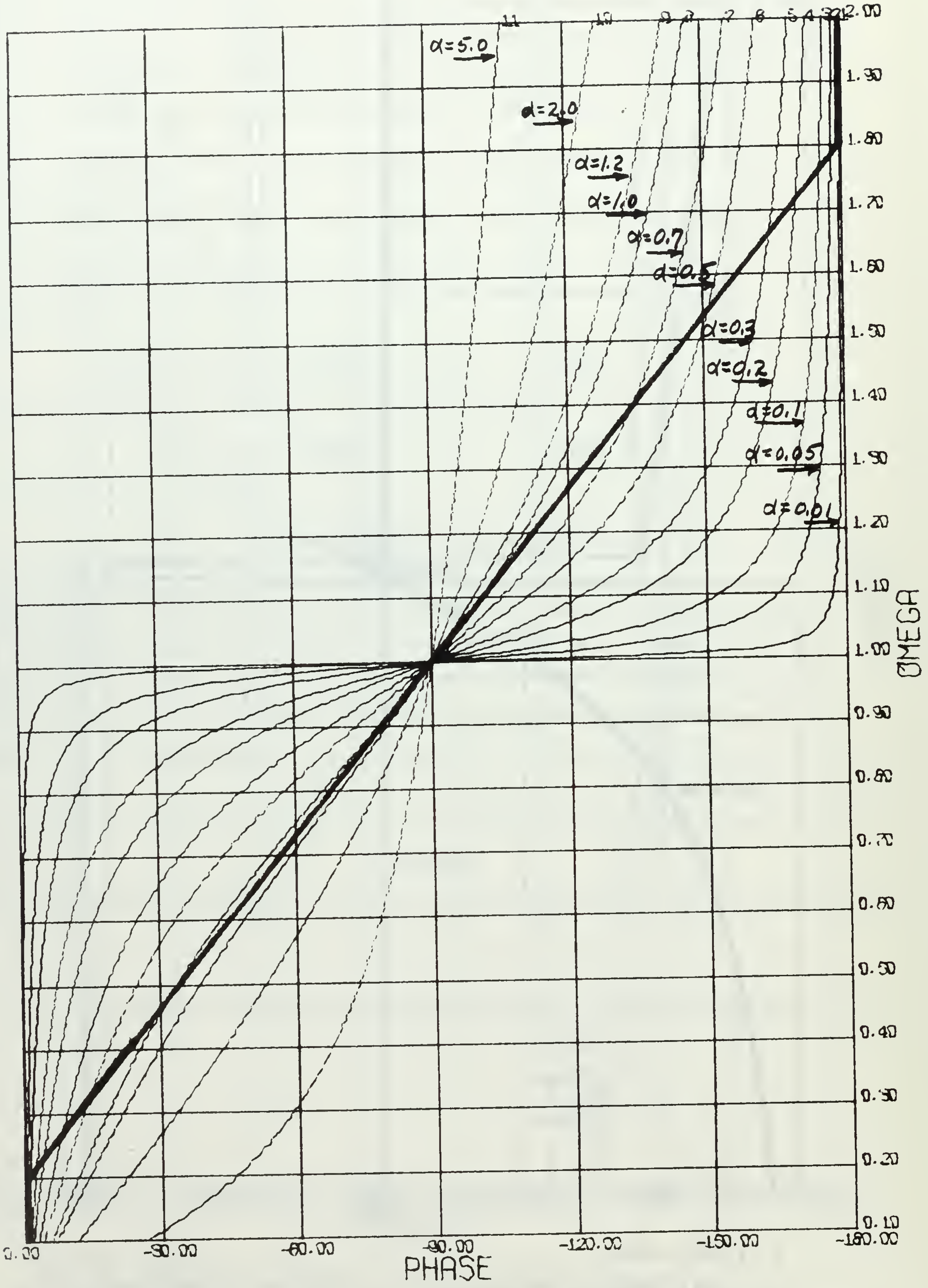


FIGURE 41.
R.A.NICHØLS,PHASE VS ØMEGA.PARAM=ALPHA=R
PHASE DESIGN WITH LØWPASS FILTER. L=C=1.0

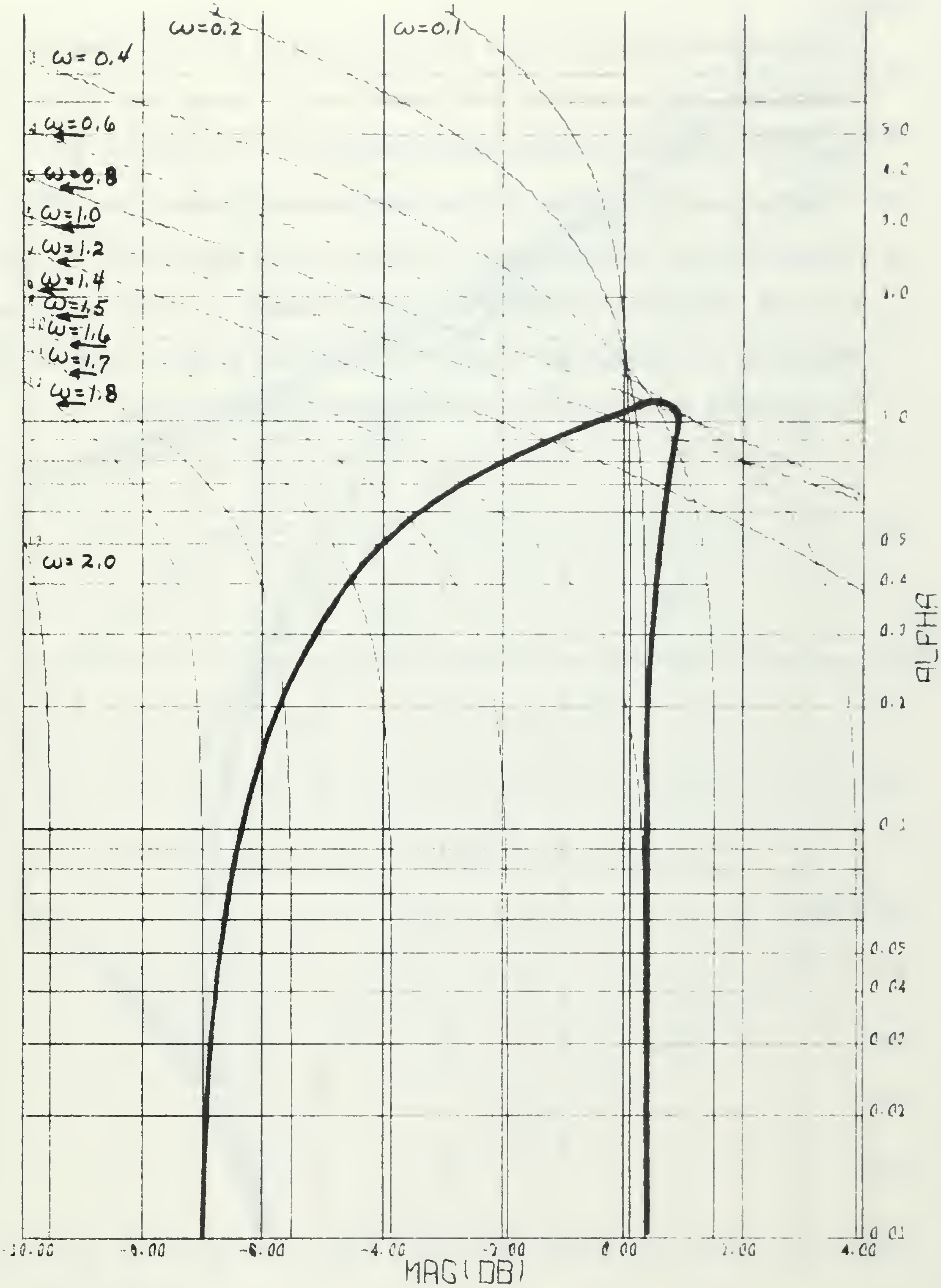


FIGURE 42.
R.A.NICHOLS, ALPHA(R) VS MAG(DB), PARAM=OMEGA
PHASE DESIGN WITH LOWPASS FILTER, L=C=1.0

alpha is fairly flat for the lower half of the frequency range but falls off in the upper half. This may also be seen in the magnitude-versus-omega plot of Figure 43.

The basic lowpass filter may be designed for a 180-degree linear phase shift if the output magnitude is not part of the specifications. If, however, a flat magnitude response is also important, the maximum phase shift would be limited to 90 degrees for this frequency range.

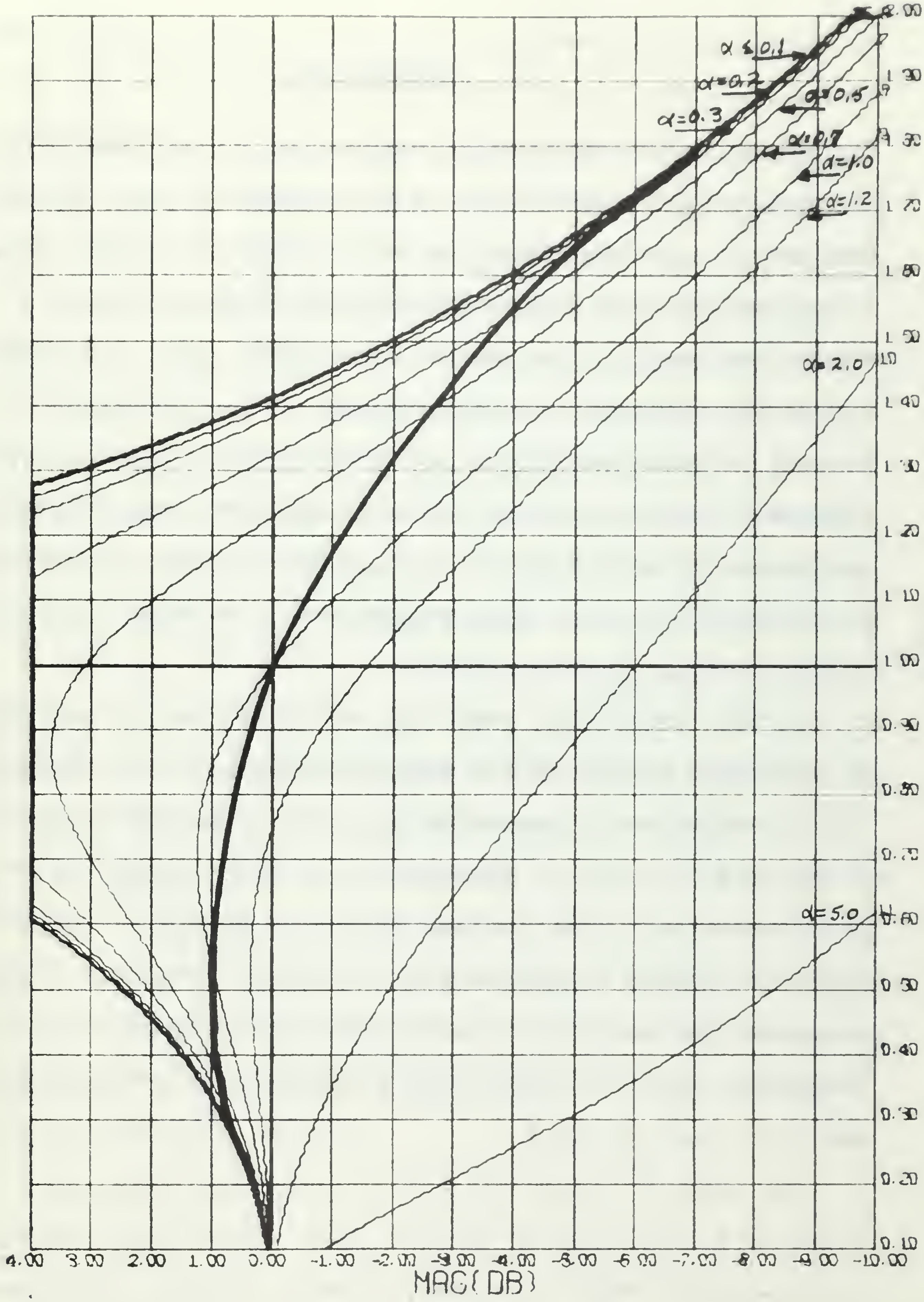


FIGURE 43.
R.A.NICHOLS.MAG(DB) VS ØMEGA.PARAM=ALPHA=R
PHASE DESIGN WITH LØWPASS FILTER. L=C=1.0

## IV. CONCLUSIONS

Filters with nonlinear elements may be designed using the frequency response form of the parameter plane method. The method is a graphical one which might be thought of as a counterpart to the describing-function method used in analytical design. Parameter-plane plots furnish the magnitude and frequency characteristics of the nonlinear element so that the filter may be designed to meet specifications. This method has the advantage of presenting the designer with a picture of the possible circuit conditions and thereby allowing value judgements to be made concerning the particular response chosen.

On the other hand, this form of filter design has the disadvantage common to all graphical methods. As the number of variables increases so does the number of graphs needed and the rate of growth of numbers of graphs is much faster than that for the number of variables. The point is finally reached where the number of graphs needed surpasses the ability of the designer to interpret and integrate them. At this point a hybrid form of graphical-analytics must be used.

The basic building block for filters with nonlinear elements is the lowpass filter. With a nonlinear R having a four-decade range of values, it will provide a stopband attenuation of 89 dB and a passband with a maximum rise of less than 1.5 dB. Adding a nonlinear L or C with a

two-decade range of values will increase the stopband attenuation to 120 dB. Multiple filter sections may be concatenated to reach even higher levels of attenuation. If the L and C elements are interchanged, the lowpass filter is transformed into a highpass filter. A bandpass filter may be formed by concatenating low- and highpass sections. The band-reject filter may be generated from the bandpass form. The lowpass filter may also be designed for a linear phase response. A linear 180-degree phase response is available for cases where a flat output magnitude is not necessary, or a linear 90-degree response where a reasonably flat output magnitude is specified.

Several areas exist for future investigation. The first one is that of the physical realization of these nonlinear filters, primarily the lowpass filter. Another is the investigation into the response of the filter to complex wave shapes. A third area is the extension of the method to include more complicated circuit configurations where the question of the optimum mix of nonlinear elements does not have an apparent answer. In addition, the use of feedback and feedforward paths should be investigated as response-shaping methods.

# APPENDIX A. COMPUTER PROGRAM INFORMATION

The two computer programs are written in the common FORTRAN IV language for use on the IBM-360 model 67 computer with a CALCOMP model 765 plotter for graphics. This is the installation at the Naval Postgraduate School. Documentation including data-card format information and internal section headings is written into each program. Both programs are capable of expansion beyond the present limitations of tenth-order transfer function and third-order alpha/beta parameters with 16 curves per graph and 900 points per curve. Linear, semi-log and log-log plots are available.

## 1. PARTF1 PROGRAM

PARTF1 provides graphs of alpha, beta, and omega (ordinate) versus magnitude (abscissa). Numerator and denominator polynomials in omega squared are formed from the s-domain transfer function and magnitude squared determined from their quotient. Graph points are generated by incrementing the desired ordinate parameter through the selected range of values while the other two parameters are held constant. The curve-family parameter is then set at the value for the next curve and the process repeated. Phase values are determined along with magnitude whenever the incremented parameter is omega. These phase values are available for plotting along the abscissa against omega.

Subroutines used with PARTF1 are GRPLOT, ABVAR and INTTFN. GRPLOT provides the plotting package for the program. The other two are used with the circuit simulation feature.

ABVAR is used to enter the FORTRAN statements describing the nonlinear parameter characteristics. Alpha and beta are initialized in the main program at each incrementation of omega, a solution made for magnitude and the information passed to this subroutine. The values of alpha, beta, magnitude and omega are then available for logic testing to determine the required alpha and beta values for that circuit condition. Flag KNR will cause a recycling of the magnitude solution using the new values of alpha and beta until the change in alpha is less than EPSA and in beta less than EPSB. KK1 is available as a loop counter. It is initialized whenever alpha and beta are initialized.

In cases where one of the parameters is not a function of the output transfer function, INTTFN is used. It is called by ABVAR when MDATA is set to one in the data set. The subroutine reads in additional data describing the internal transfer function and uses the ABVAR information to compute magnitude. This value, X2, is then passed back to ABVAR for use in determining parameter values.

The storage requirements of PARTF1 are set primarily by the "DIMENSION PHASE (16,901), YY(16,901)" card. The number "16" represents the maximum number of curve parameters or

voltage input values used while "901" is the maximum number of computation points plus one. Using these values, the program requires 205K plus 9K for the plotting package and about 19K for operations. This comes to about 233K. This may be reduced considerably by changing the statement to reflect the actual number required. For instance, if only six curves were to be plotted, the storage requirements would be reduced by about 72K (2 X 10 X 901 X 4). If only 500 points were used for each of the six curves, a further reduction in storage of about 19K would be gained.

As expected, the number of curves and plotting points also affects the run time. Compile time is about 23 seconds and the time for the link step about 1.5 seconds. A rough rule of thumb for go time is four seconds per 900-point curve. A six-curve run of 900 points per curve would have an execute time of around 48.5 seconds or about 50 seconds if a phase plot were also made.

Figure 44 is a repeat of Figure 6 using only 300 points per curve. Figure 45 uses only 100 points per curve. For design graphs of this size it would be wasteful in both storage and run time if 900 points per curve were demanded.

2. PARTF2 PROGRAM

PARTF2 provides graphs of alpha and beta versus omega, and alpha versus beta. As in the first program the input s-domain transfer function is separated into numerator and denominator functions which are individually transformed into the omega-squared domain. In PARTF2, however, the

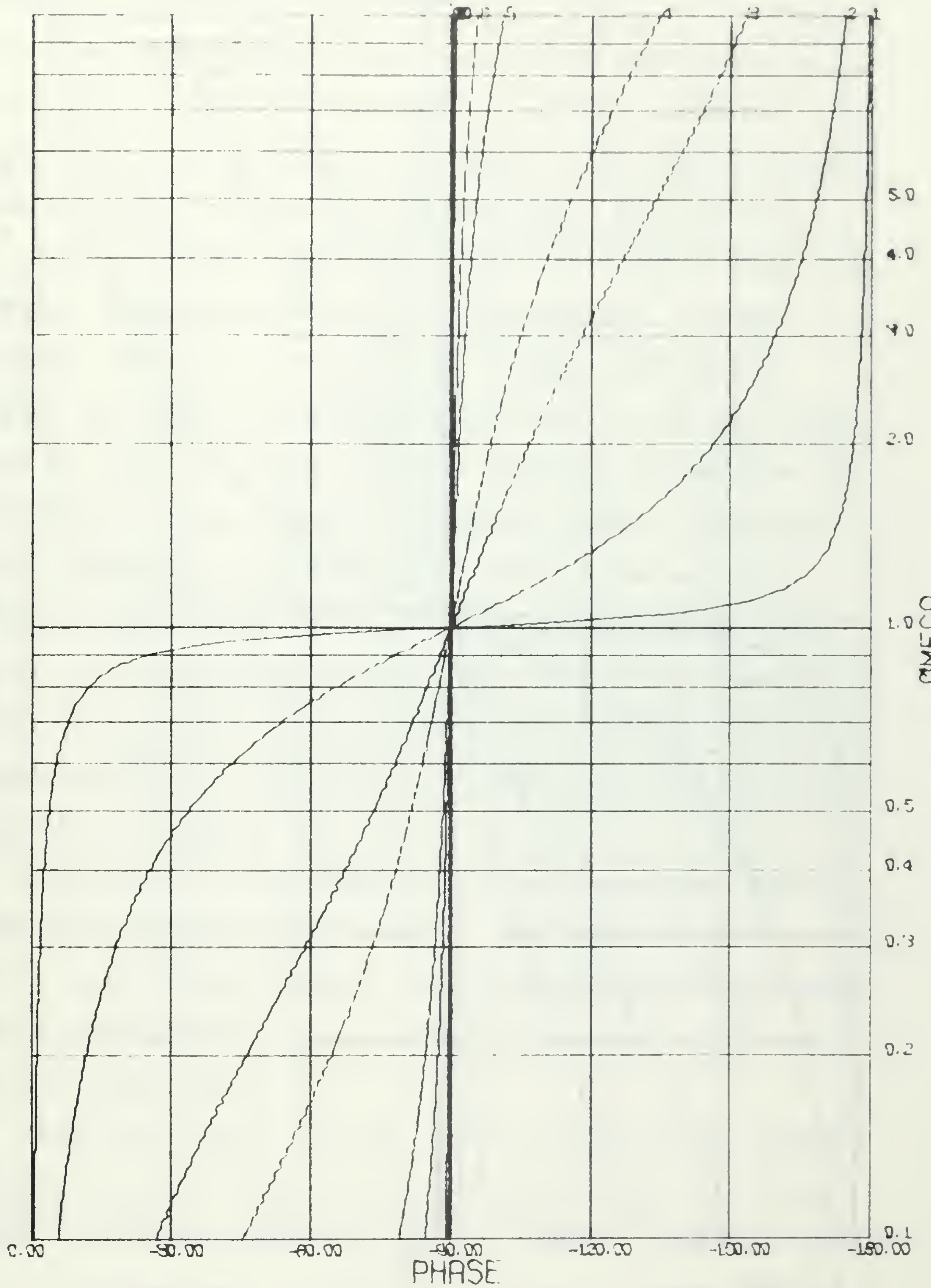


FIGURE 44.
R.A.NICHOLS. PHASE PLOT. PARAMETER=ALPHA
LOWPASS FILTER. L=C=1.0. NRPNTS=300

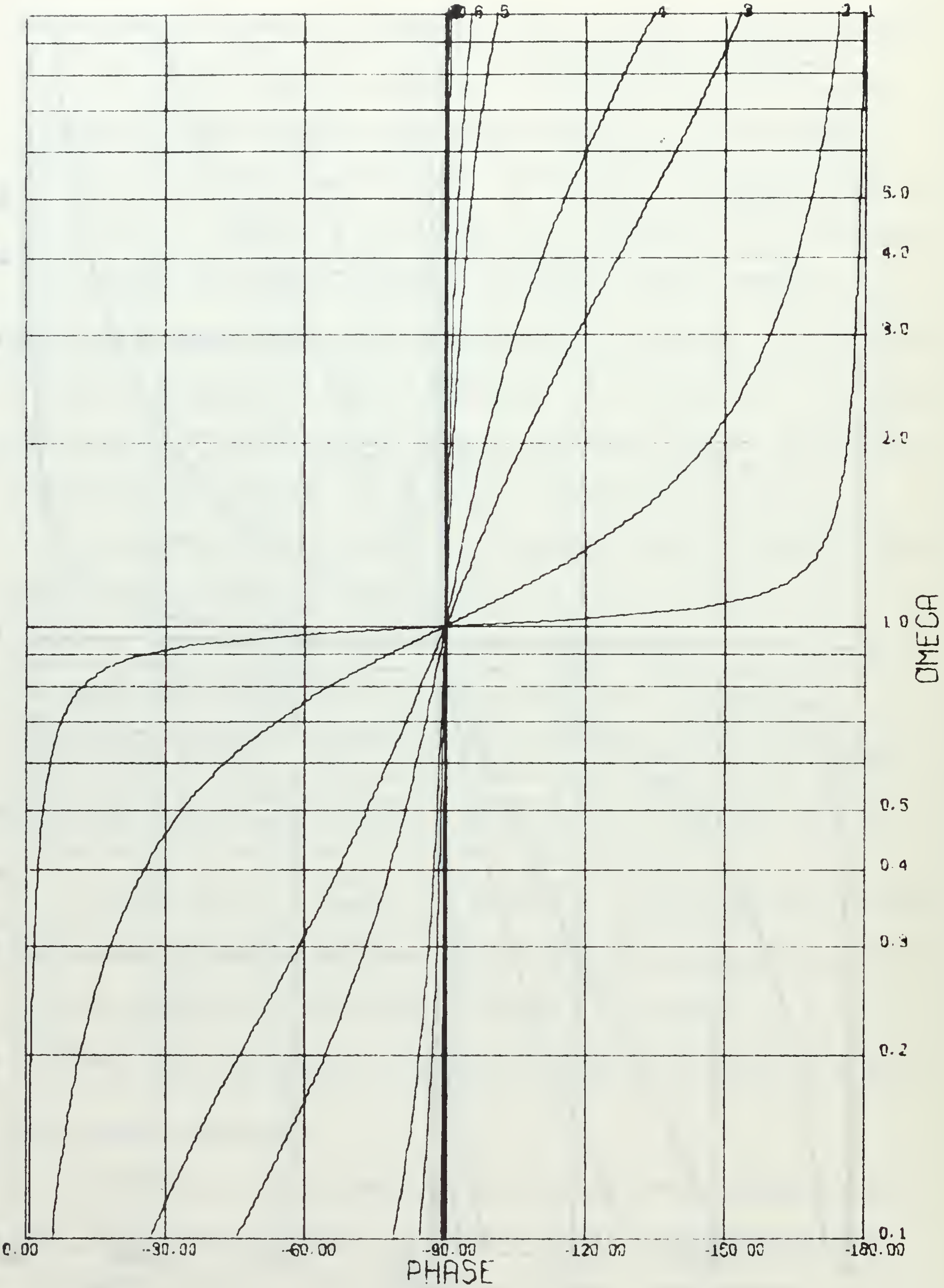


FIGURE 45.
R.A.NICHOLS. PHASE PLOT. PARAMETER=ALPHA
LOWPASS FILTER, L=C=1.0. NRPNTS=100

denominator is then multiplied by the square of the magnitude and subtracted from the numerator to form a single polynomial in terms of the unknown ordinate parameter. The real roots of this polynomial together with the incremented abscissa values generate the graph points for the curve.

The subroutines used with PARTF2 are GRPLOT, POLRT2 and POLRT3. GRPLOT provides the plotting package for the program. The other two are root-finding subroutines. POLRT2 is used when the polynomial order is two and POLRT3 when the order is three or more. Exponential overflow and underflow may occur in the high-order terms of the polynomial in POLRT3. This will not affect accuracy. The ERRSET command cancels the overflow and underflow error print order and prevents the program from being prematurely ended when a given number of them have occurred. POLRT3 is a modified POLRT routine from the IBM System/360 Scientific Subroutine Package.

Contrary to PARTF1 there is no one predominant factor setting the storage requirements. The limiting case of each factor contributes equally. Including the plotting package and operating space requirements, the storage requirement comes to about 148K.

PARTF2 run times are much longer than those for PARTF1. Forming the polynomial, solving for the roots, selecting roots, etc. multiplies the run time over that for the simple quotient procedure of PARTF1 by a factor of seven. The go

time rule of thumb is 28 seconds per 900-point curve. The compile, link and edit time is about 23 seconds. Thus the estimated run time for eight 900-point curves would be about 247 seconds (8 X 28 + 23). It pays to use fewer curves and fewer points with PARTF2.

PARTF1   PROGRAM

```
C
C    PARTF1, PARAMETER PLANE TRANSFER FUNCTION PLOTTING ROUTINE NUMBER 1,
C    ALPHA, BETA, OMEGA, PHASE VERSUS MAGNITUDE. PROGRAM INPUT IS THE
C    CIRCUIT TRANSFER FUNCTION IN THE S-PLANE, T(S). THE PROGRAM COMPUTES
C    T(JW) AND SOLVES FOR M**2. T(S) IS LIMITED TO TENTH ORDER. THE
C    MAXIMUM EXPONENT VALUE FOR ALPHA AND BETA IS THREE (SEE FORMAT
C    CARD NUMBER NINE).
C
C
C  CARD FORMAT   SYMBOL               SYMBOL USAGE
C
C    1    8I10     NRUN    NUMBER OF RUNS DESIRED. EACH RUN MUST HAVE A FULL
C                          SET OF DATA CARDS.
C
C    2    8I10    PLOTNR   PLOT NUMBER (Y-AXIS VS X-AXIS)
C                          1=ALPHA VS MAGNITUDE,BETA FIXED,OMEGA PARAMETER.
C                          2=BETA VS MAGNITUDE,ALPHA FIXED,OMEGA PARAMETER.
C                          3=OMEGA VS MAGNITUDE,BETA FIXED,ALPHA PARAMETER.
C                          4=OMEGA VS MAGNITUDE,ALPHA FIXED,BETA PARAMETER.
C                          5=OMEGA VS MAGNITUDE,ALPHA AND/OR BETA FUNCTIONS
C                            OF OUTPUT MAGNITUDE AND/OR RADIAN FREQUENCY.
C                 (NOTE:   PHASE VS MAGNITUDE IS AVAILABLE WITH PLOTNR 3,4,5.
C                          SEE IGRAPH.)
C                 NORNUM   ORDER OF NUMERATOR POLYNOMIAL (MAX=10).
C                 NORDEN   ORDER OF DENOMINATOR POLYNOMIAL (MAX=10).
C                 NPCURV   NUMBER OF PARAMETER VALUES (1-16).
C                          (FOR PLOTNR 5 SET NPCURV=1)
C      NOTE: THE DIMENSION PHASE(16,901),YY(16,901) CARD MAY BE CHANGED
C                          TO REDUCE THE STORAGE REQUIRED. THE FIRST NUMBER
C                          MUST BE .GE. NPCURV.
C                 NRPNTS   NUMBER OF COMPUTATIONS BETWEEN MAX AND MINVALUES.
C                          IT EQUALS THE NUMBER OF POINTS PLOTTED PER CURVE
C                          (1-900). IF SET=0, NRPNTS=900.
C                 IWRITE   NUMBER OF COMPUTATIONS BETWEEN PRINTOUTS (1-900).
C                          IF SET=0, NO COMPUTATION RESULTS ARE PRINTED.
C                 NVINPT   NUMBER OF VOLTAGE INPUTS (1-16). USED WITH
C                          PLOTNR 3,4,5 WHEN E-IN IS DIFFERENT FROM 1.0.
C                          IF SET=0, E-IN IS EQUAL TO 1.0.
C
C    3    8I10    IGRAPH   GRAPH ORDER.
C                          -1= NO GRAPHS DESIRED.
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C                      0= GRAPHS ARE DESIRED.                                TF1041
C                      1= GRAPH OF PHASE VS OMEGA DESIRED. USED WITH         TF1042
C                     PLOTNR 3,4,5.                                          TF1043
C             ALPSCL  ALPHA OUTPUT AXIS SCALE CODE. USED WITH PLOTNR 1.      TF1044
C                     0=LOG SCALE.                                           TF1045
C                     1=LINEAR SCALE.                                        TF1046
C             BETSCL  BETA OUTPUT AXIS SCALE CODE. USED WITH PLOTNR 2.       TF1047
C                     0=LOG SCALE.                                           TF1048
C                     1=LINEAR SCALE.                                        TF1049
C             MAGSCL  MAGMIN,MAGMAX AND MAGNITUDE OUTPUT SCALE CODE.         TF1050
C                     0=DB VALUES.                                           TF1051
C                     1=ACTUAL VALUES.                                       TF1052
C                     2=SQUARED VALUES.                                      TF1053
C             OMGSCL  OMEGA OUTPUT AXIS SCALE CODE. USED WITH PLOTNR 3,      TF1054
C                     4,5.                                                   TF1055
C                     0=LOG SCALE.                                           TF1056
C                     1=LINEAR SCALE.                                        TF1057
C              MDATA  SUBROUTINE INTTEN ACTIVATION. USED WITH PLOTNR         TF1058
C                     5 WHEN PARAMETER IS A FUNCTION OF AN INTERNAL          TF1059
C                     TRANSFER FUNCTION. SEE THE SUBROUTINE FOR DETAILS      TF1060
C                                                                            TF1061
C    4   8E10.3        YHIGH  HEIGHT OF THE VERTICAL AXIS, Y-AXIS, OF THE GRAPH  TF1062
C                     IN INCHES. FOR TOTAL PAGE HEIGHT ADD 1.0 INCHES        TF1063
C                     FOR GRAPH SCALES, AXIS LABEL AND GRAPH TITLING.        TF1064
C              XWIDE  WIDTH OF THE HORIZONTAL AXIS, X-AXIS, OF THE           TF1065
C                     GRAPH IN INCHES. FOR TOTAL PAGE WIDTH ADD 0.5          TF1066
C                     INCHES FOR GRAPH SCALES AND AXIS LABEL. TOTAL          TF1067
C                     WIDTH IS LIMITED BY PAPER SIZE, USUALLY 9.0 INCHES     TF1068
C               YUPL  NUMBER OF UNITS PER LINE ON THE VERTICAL SCALE.        TF1069
C                     IF SET=0, IT WILL BE SCALED AUTOMATICALLY TO           TF1070
C                     (YMAX-YMIN)/YHIGH. IT IS NOT USED WITH LOG SCALES.     TF1071
C               XUPL  NUMBER OF UNITS PER LINE ON THE HORIZONTAL SCALE.      TF1072
C                     IF SET=0, IT WILL BE SCALED AUTOMATICALLY TO           TF1073
C                     (XMAX-XMIN)/XWIDE. IT IS NOT USED WITH LOG SCALES.     TF1074
C             PHSMIN  MINIMUM VALUE FOR PHASE. ALL PHASE VALUES ARE          TF1075
C                     CONSIDERED NEGATIVE (-360.0 .LE. PHASE .LE. 0.0).      TF1076
C                     IF SET=0, PHSMIN=-360.0.                               TF1077
C             PHSMAX  MAXIMUM VALUE FOR PHASE.                               TF1078
C             PHSUPL  NUMBER OF DEGREES PER SCALE MARKING. IF SET=0,         TF1079
C                     IT WILL BE AUTOMATICALLY SCALED TO 45.0 DEGREES        TF1080
C                     PER LINE.                                              TF1081
C                                                                            TF1082
C    5   8E10.3       ALPMIN  MINIMUM VALUE FOR ALPHA.                       TF1083
C             ALPMAX  MAXIMUM VALUE FOR ALPHA.                               TF1084
C             BETMIN  MINIMUM VALUE FOR BETA.                                TF1085
C             BETMAX  MAXIMUM VALUE FOR BETA.                                TF1086
C             MAGMIN  MINIMUM VALUE FOR MAGNITUDE. SEE MAGSCL.               TF1087
C             MAGMAX  MAXIMUM VALUE FOR MAGNITUDE. SEE MAGSCL.               TF1088
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C              OMGMIN   MINIMUM VALUE FOR OMEGA.                               TF1089
C              OMGMAX   MAXIMUM VALUE FOR OMEGA.                               TF1090
C      NOTE: IF AXIS SCALE CODE IS LOG, MIN.GE.0.001, MAX.LE.10000.0.          TF1091
C                                                                              TF1092
C    6   8E10.3   SETALP   VALUE OF ALPHA WHEN USED AS A CONSTANT.             TF1093
C                 SETBET   VALUE OF BETA WHEN USED AS A CONSTANT.              TF1094
C                          (USED AS INITIAL ALPHA/BETA VALUES WHEN PLOTNR=5)   TF1095
C                                                                              TF1096
C    7   8E10.3   VINPUT   VOLTAGE INPUT VALUES (VINPUT*1.0) USED WITH         TF1097
C                          PLOTNR 3,4,5. OMIT IF PLOTNR =1,2 OR NVINPT=0.      TF1098
C                          USE TWO CARDS IF NVINPT .GT. 8.                     TF1099
C                                                                              TF1100
C    8   8E10.3   CURPAR   CURVE PARAMETER VALUES. USE TWO CARDS IF NPCURV     TF1101
C                          .GT.8. OMIT IF PLOTNR=5.                            TF1102
C                                                                              TF1103
C    9   8E10.3     AN     NUMERATOR COEFFICIENTS IN ASCENDING ORDER OF        TF1104
C                          S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S   TF1105
C                          EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER.   TF1106
C                          (1)  *1.0,     *B,        *B**2,       *B**3,       TF1107
C                               *A,       *A*B,      *A*B**2,     *A*B**3.     TF1108
C                          (2)  *A**2,    *A**2*B,   *A**2*B**2,  *A**2*B**3,  TF1109
C                               *A**3,    *A**3*B,   *A**3*B**2,  *A**3*B**3.  TF1110
C                                                                              TF1111
C   10   8E10.3     AD     DENOMINATOR COEFFICIENTS IN ASCENDING ORDER OF      TF1112
C                          S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S   TF1113
C                          EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER.   TF1114
C                          SAME FORMAT AS CARD NUMBER 9 (NUMERATOR COEFF).     TF1115
C                                                                              TF1116
C   11    14A4    GTITL1   GRAPH TITLE LINE NUMBER ONE.                        TF1117
C                                                                              TF1118
C   12    14A4    GTITL2   GRAPH TITLE LINE NUMBER TWO.                        TF1119
C                                                                              TF1120
C   13    14A4    GTITL3   GRAPH TITLE LINE NUMBER THREE.                      TF1121
C                                                                              TF1122
C   14    14A4    GTITL4   PHASE GRAPH TITLE LINE NUMBER ONE.                  TF1123
C                                                                              TF1124
C   15    14A4    GTITL5   PHASE GRAPH TITLE LINE NUMBER TWO.                  TF1125
C                                                                              TF1126
C   16    14A4    GTITL6   PHASE GRAPH TITLE LINE NUMBER THREE.                TF1127
C                                                                              TF1128
C     NOTE: OMIT CARDS 11-16 IF IGRAPH=-1. OMIT CARDS 13-16 IF IGRAPH=0.       TF1129
C                                                                              TF1130
C                                                                              TF1131
C****************** PROGRAM FOLLOWS *************************************      TF1132
      INTEGER PLOTNR,ALPSCL,BETSCL,OMGSCL,ZSCALE                               TF1133
      REAL MAGMAX,MAGMIN                                                       TF1134
      DIMENSION A(11),C(11),CN(11),CD(11),PAN(11),PAD(11)                      TF1135
      DIMENSION CURPAR(16),VINPUT(16)                                          TF1136
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      DIMENSION GTITL1(14),GTITL2(14),GTITL3(14),GTITL4(14),GTITL5(14),
     *          GTITL6(14)
      DIMENSION ALPHA(901),BETA(901),XMAG(901),OMEGA(901),X(901),Y(901),
     *          MAXIP(901),PHSDEG(901),ZVAL(901)
      DIMENSION AA(11,16),AD(11,16),AN(11,16)
      DIMENSION PHASE(16,901),YY(16,901)
      COMMON MAGSCL,MABVAR,EPSLOG,KNR,KK1,MDATA
      COMMON GTITL1,GTITL2,GTITL3,GTITL4,GTITL5,GTITL6
      COMMON XMAX,XMIN,ZMAX,ZMIN,MSCALE,XSCALE,YSCALE,ZSCALE,ZLOGMN
      COMMON X,Y,XUPL,YUPL,XWIDE,YHIGH,PLOTNR,IC,IP
    1 FORMAT(1H1)
    5 FORMAT(14A4)
    6 FORMAT(10X,14A4)
    7 FORMAT(8I10)
    8 FORMAT(10X,8I10)
    9 FORMAT(8E10.3)
   10 FORMAT(10X,1P8E12.3)
C****************** READ IN DATA ***************************************
      READ(5,7) NRUN
      IRUN=1
   19 READ(5,7) PLOTNR,NORNUM,NORDEN,NPCURV,NRPNTS,IWRITE,NVINPT
      NNP1=NORNUM+1
      NDP1=NORDEN+1
      IF(NRPNTS.EQ.0) NRPNTS=900
      NRPTM1=NRPNTS-1
      IPNR=1
      EPSLOG=1.0E-50
      READ(5,7) IGRAPH,ALPSCL,BETSCL,MAGSCL,OMGSCL,MDATA
      MABVAR=1
      READ(5,9) YHIGH,XWIDE,YUPL,XUPL,PHSMIN,PHSMAX,PHSUPL
      READ(5,9) ALPMIN,ALPMAX,BETMIN,BETMAX,MAGMIN,MAGMAX,OMGMIN,OMGMAX
      IF((PLOTNR.GE.3).OR.(YUPL.NE.0.0)) GO TO 20
      IF(PLOTNR.EQ.1) YUPL=(ALPMAX-ALPMIN)/YHIGH
      IF(PLOTNR.EQ.2) YUPL=(BETMAX-BETMIN)/YHIGH
   20 IF(XUPL.EQ.0.0) XUPL= (MAGMAX-MAGMIN)/XWIDE
      IF(IGRAPH.LE.0) GO TO 21
      IF(PHSMIN.EQ.0.0) PHSMIN=-360.0
      IF(PHSUPL.EQ.0.0) PHSUPL=45.0
   21 READ(5,9) SETALP,SETBET
      VINPUT(1)=1.0
      IF((NVINPT.EQ.0).OR.(PLOTNR.LE.2)) GO TO 22
      READ(5,9) (VINPUT(I),I=1,NVINPT)
      GO TO 23
   22 READ(5,9) (CURPAR(I),I=1,NPCURV)
   23 DO 24 JN=1,NNP1
   24 READ(5,9) (AN(JN,I),I=1,16)
      DO 26 JD=1,NDP1
   26 READ(5,9) (AD(JD,I),I=1,16)
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      IF(IGRAPH.EQ.-1) GO TO 30                                          TF1185
      READ(5,5) (GTITL1(I),I=1,14)                                       TF1186
      READ(5,5) (GTITL2(I),I=1,14)                                       TF1187
      READ(5,5) (GTITL3(I),I=1,14)                                       TF1188
      IF((IGRAPH.EQ.0).OR.(PLOTNR.LE.2)) GO TO 30                        TF1189
      READ(5,5) (GTITL4(I),I=1,14)                                       TF1190
      READ(5,5) (GTITL5(I),I=1,14)                                       TF1191
      READ(5,5) (GTITL6(I),I=1,14)                                       TF1192
C****************** WRITE INPUT DATA ************************************TF1193
   30 WRITE(6,1)                                                         TF1194
      IF(IRUN.GT.1) GO TO 32                                             TF1195
      WRITE(6,31) NRUN                                                   TF1196
   31 FORMAT(14X,'NRUN=',I3)                                             TF1197
   32 WRITE(6,33)                                                        TF1198
   33 FORMAT(/,16X,'PLOTNR',4X,'NORNUM',4X,'NORDEN',4X,'NPCURV',4X,'NRPN TF1199
     *TS',4X,'IWRITE',4X,'NVINPT')                                       TF1200
      WRITE(6,8) PLOTNR,NORNUM,NORDEN,NPCURV,NRPNTS,IWRITE,NVINPT        TF1201
      WRITE(6,35)                                                        TF1202
   35 FORMAT(/,16X,'IGRAPH',4X,'ALPSCL',4X,'BETSCL',4X,'MAGSCL',4X,'OMGS TF1203
     *CL',5X,'MDATA')                                                    TF1204
      WRITE(6,8) IGRAPH,ALPSCL,BETSCL,MAGSCL,OMGSCL,MDATA                TF1205
      WRITE(6,37)                                                        TF1206
   37 FORMAT(/,15X,'YHIGH',7X,'XWIDE',7X,'YUPL',8X,'XUPL',7X,'PHSMIN',6X TF1207
     *,'PHSMAX',5X,'PHSUPL')                                             TF1208
      WRITE(6,10) YHIGH,XWIDE,YUPL,XUPL,PHSMIN,PHSMAX,PHSUPL             TF1209
      WRITE(6,39)                                                        TF1210
   39 FORMAT(/,14X,'ALPMIN',6X,'ALPMAX',6X,'BETMIN',6X,'BETMAX',6X,'MAGM TF1211
     *IN',6X,'MAGMAX',6X,'OMGMIN',6X,'OMGMAX')                           TF1212
      WRITE(6,10)ALPMIN,ALPMAX,BETMIN,BETMAX,MAGMIN,MAGMAX,OMGMIN,OMGMAX TF1213
      WRITE(6,41)                                                        TF1214
   41 FORMAT(/,14X,'SETALP',6X,'SETBET')                                 TF1215
      WRITE(6,10) SETALP,SETBET                                          TF1216
      IF((NVINPT.EQ.0).OR.(PLOTNR.LE.2)) GO TO 43                        TF1217
      WRITE(6,42)                                                        TF1218
   42 FORMAT(//,10X,'THE FOLLOWING INPUT VOLTAGE VALUES WILL BE USED:',/ TF1219
     *)                                                                  TF1220
      WRITE(6,10) (VINPUT(KV),KV=1,NVINPT)                               TF1221
   43 GO TO (48,48,44,46,50),PLOTNR                                      TF1222
   44 WRITE(6,45)                                                        TF1223
   45 FORMAT(//,10X,'THE VALUES FOR THE ALPHA PARAMETER CURVES ARE',/)   TF1224
      GO TO 52                                                           TF1225
   46 WRITE(6,47)                                                        TF1226
   47 FORMAT(//,10X,'THE VALUES FOR THE BETA PARAMETER CURVES ARE',/)    TF1227
      GO TO 52                                                           TF1228
   48 WRITE(6,49)                                                        TF1229
   49 FORMAT(//,10X,'THE VALUES FOR THE OMEGA PARAMETER CURVES ARE',/)   TF1230
      GO TO 52                                                           TF1231
   50 WRITE(6,51)                                                        TF1232
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51 FORMAT(//,10X,'PARAMETER CURVES ARE NOT USED WITH PLOTNR 5. ALPHA    TF1233
  *AND/OR BETA ARE FUNCTIONS OF OUTPUT MAGNITUDE AND/OR RADIAN FREQUE   TF1234
  *NCY.'/)                                                              TF1235
   GO TO 53                                                             TF1236
52 WRITE(6,10) (CURPAR(I),I=1,NPCURV)                                   TF1237
53 WRITE(6,54)                                                          TF1238
54 FORMAT(//,10X,'NUMERATOR FUNCTION',/)                                TF1239
55 FORMAT(3X,'S**',I2,' * (',1P1E10.3,'*1.0       + ',1P1E10.3,'*B      TF1240
  *       + ',1P1E10.3,'*B**2       + ',1P1E10.3,'*B**3       + ',/,12X, TF1241
  *1P1E10.3,'*A           + ',1P1E10.3,'*A*B        + ',1P1E10.3,'*A*B** TF1242
  *2     + ',1P1E10.3,'*A*B**3      + ',/,12X,1P1E10.3,'*A**2       + ',1 TF1243
  *P1E10.3,'*A**2*B      + ',1P1E10.3,'*A**2*B**2 + ',1P1E10.3,'*A**2*B TF1244
  ***3 + ',/,12X,1P1E10.3,'*A**3        + ',1P1E10.3,'*A**3*B     + ',1P TF1245
  *1E10.3,'*A**3*B**2 + ',1P1E10.3,'*A**3*B**3)',/)                    TF1246
   DO 56 JN=1,NNP1                                                      TF1247
   JNM1=JN-1                                                            TF1248
56 WRITE(6,55) JNM1,(AN(JN,I),I=1,16)                                   TF1249
   WRITE(6,58)                                                          TF1250
58 FORMAT(//,10X,'DENOMINATOR FUNCTION',/)                              TF1251
   DO 59 JD=1,NDP1                                                      TF1252
   JDM1=JD-1                                                            TF1253
59 WRITE(6,55) JDM1,(AD(JD,I),I=1,16)                                   TF1254
   IF(IGRAPH.EQ.-1) GO TO 69                                            TF1255
   WRITE(6,60)                                                          TF1256
60 FORMAT(//,10X,'THE GRAPH WILL BE TITLED',/)                          TF1257
   WRITE(6,6) (GTITL1(IT),IT=1,14)                                      TF1258
   WRITE(6,6) (GTITL2(IT),IT=1,14)                                      TF1259
   WRITE(6,6) (GTITL3(IT),IT=1,14)                                      TF1260
   IF((IGRAPH.EQ.0).OR.(PLOTNR.LE.2)) GO TO 69                          TF1261
   WRITE(6,62)                                                          TF1262
62 FORMAT(/,10X,'THE PHASE GRAPH WILL BE TITLED',/)                     TF1263
   WRITE(6,6) (GTITL4(IT),IT=1,14)                                      TF1264
   WRITE(6,6) (GTITL5(IT),IT=1,14)                                      TF1265
   WRITE(6,6) (GTITL6(IT),IT=1,14)                                      TF1266
C***************** SET UP GRAPH SCALES ********************************** TF1267
69 IF(PLOTNR-2) 70,71,72                                                TF1268
70 ZMAX=ALPMAX                                                          TF1269
   ZMIN=ALPMIN                                                          TF1270
   ZSCALE=ALPSCL                                                        TF1271
   GO TO 73                                                             TF1272
71 ZMAX=BETMAX                                                          TF1273
   ZMIN=BETMIN                                                          TF1274
   ZSCALE=BETSCL                                                        TF1275
   GO TO 73                                                             TF1276
72 ZMAX=OMGMAX                                                          TF1277
   ZMIN=OMGMIN                                                          TF1278
   ZSCALE=OMGSCL                                                        TF1279
73 XMAX=MAGMAX                                                          TF1280
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      XMIN=MAGMIN
      MSCALE=MAGSCL
      CALL GRPLOT(1)
C****************** MAIN LOOP GENERATION.*******************************
   80 IP=0
      ID=1
      KV=1
      IN=1
      IW=1
      KK1=1
   81 AL=SETALP
      BE=SETBET
   82 IF(ZSCALE.EQ.0) GO TO 84
      ZINC=(ZMAX-ZMIN)/NRPTM1
   83 ZVAL(IN)=ZMIN+(IN-1)*ZINC
      GO TO 86
   84 TLZMAX=ALOG10(ZMAX)
      TLZMIN=ALOG10(ZMIN)
      ZINC=(TLZMAX-TLZMIN)/NRPTM1
   85 IF(PLOTNR.GT.2) GO TO 86
      TLZVAL=TLZMIN+(IN-1)*ZINC
      ZVAL(IN)=10.0**TLZVAL
   86 GO TO (87,88,89,90,91),PLOTNR
   87 AL=ZVAL(IN)
      ALPHA(IN)=AL
      VAL=AL
      GO TO 91
   88 BE=ZVAL(IN)
      BETA(IN)=BE
      VAL=BE
      GO TO 91
   89 AL=CURPAR(ID)
      GO TO 91
   90 BE=CURPAR(ID)
C****************** NUMERATOR AND DENOMINATOR COEFFICIENT CALCULATION***
   91 IKOUNT=1
      N=NNP1
      DO 92 I=1,N
      DO 92 J=1,16
   92 AA(I,J)=AN(I,J)
   93 DO 94 I=1,N
   94 A(I)=AA(I,1)+AA(I,2)*BE+AA(I,3)*BE**2+AA(I,4)*BE**3+AA(I,5)*AL+AA(
     *I,6)*AL*BE+AA(I,7)*AL*BE**2+AA(I,8)*AL*BE**3+AA(I,9)*AL**2+AA(I,10
     *)*AL**2*BE+AA(I,11)*AL**2*BE**2+AA(I,12)*AL**2*BE**3+AA(I,13)*AL**
     *3+AA(I,14)*AL**3*BE+AA(I,15)*AL**3*BE**2+AA(I,16)*AL**3*BE**3
      IF(N.EQ.11) GO TO 96
      NP1=N+1
      DO 95 I=NP1,11
```

```
 95 A(I)=0.0
 96 GO TO (107,106,105,104,103,102,101,100,99,98,97),N
 97 C(11)=A(11)*A(11)
 98 C(10)=A(10)*A(10)-2.0*A(9)*A(11)
 99 C(9)=A(9)*A(9)+2.0*(A(7)*A(11)-A(8)*A(10))
100 C(8)=A(8)*A(8)-2.0*(A(5)*A(11)-A(6)*A(10)+A(7)*A(9))
101 C(7)=A(7)*A(7)+2.0*(A(3)*A(11)-A(4)*A(10)+A(5)*A(9)-A(6)*A(8))
102 C(6)=A(6)*A(6)-2.0*(A(1)*A(11)-A(2)*A(10)+A(3)*A(9)-A(4)*A(8)+A(5)
   **A(7))
103 C(5)=A(5)*A(5)+2.0*(A(1)+A(9)-A(2)*A(8)+A(3)*A(7)-A(4)*A(6))
104 C(4)=A(4)*A(4)-2.0*(A(1)*A(7)-A(2)*A(6)+A(3)*A(5))
105 C(3)=A(3)*A(3)+2.0*(A(1)*A(5)-A(2)*A(4))
106 C(2)=A(2)*A(2)-2.0*A(1)*A(3)
107 C(1)=A(1)*A(1)
    IF(IKOUNT.EQ.2) GO TO 111
    DO 108 IG=1,N
108 CN(IG)=C(IG)
    DO 109 I=1,11
109 PAN(I)=A(I)
    N=NDP1
    IKOUNT=2
    DO 110 I=1,N
    DO 110 J=1,16
110 AA(I,J)=AD(I,J)
    GO TO 93
111 DO 112 IG=1,N
112 CD(IG)=C(IG)
    DO 113 I=1,11
113 PAD(I)=A(I)
C****************** MAGNITUDE/PHASE CALCULATION ***********************
    IC=ID
    IF(NVINPT.GE.1) IC=KV
    IF(PLOTNR-3) 114,122,122
114 SUMN=0.0
    DO 115 I=1,NNP1
    NWEXP=2*(I-1)
115 SUMN=SUMN+CN(I)*CURPAR(ID)**NWEXP
    SUMD=0.0
    DO 116 J=1,NDP1
    NWEXP=2*(J-1)
116 SUMD=SUMD+CD(J)*CURPAR(ID)**NWEXP
    XMAG(IN)=ABS(SUMN/SUMD)
    IF(MAGSCL.EQ.2) GO TO 120
    IF(MAGSCL.EQ.0) GO TO 118
117 XMAG(IN)=SQRT(XMAG(IN))
    GO TO 120
118 IF((XMAG(IN)-EPSLOG).GE.0.0) GO TO 119
    XMAG(IN)=-1.0E 70
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      GO TO 121                                                         TF1377
  119 XMAG(IN)=10.0*ALOG10(XMAG(IN))                                    TF1378
  120 IF((XMAG(IN).LT.MAGMIN).OR.(XMAG(IN).GT.MAGMAX)) GO TO 121        TF1379
      IP=IP+1                                                           TF1380
      X(IP)=(XMAG(IN)-MAGMIN)/XSCALE                                    TF1381
      Y(IP)=(VAL-ZMIN)/YSCALE                                           TF1382
      IF(ZSCALE.EQ.0) Y(IP)=(ALOG10(VAL)-ZLOGMN)/YSCALE                 TF1383
  121 IN=IN+1                                                           TF1384
      IF(IN.GT.NRPNTS) GO TO 141                                        TF1385
      GO TO 81                                                          TF1386
  122 IF(ZSCALE.EQ.0) GO TO 123                                         TF1387
      OMEGA(IW)=ZMIN+(IW-1)*ZINC                                        TF1388
      GO TO 124                                                         TF1389
  123 TLZVAL=TLZMIN+(IW-1)*ZINC                                         TF1390
      OMEGA(IW)=10.0**TLZVAL                                            TF1391
  124 SUMN=0.0                                                          TF1392
      DO 125 I=1,NNP1                                                   TF1393
      NWEXP=2*(I-1)                                                     TF1394
  125 SUMN=SUMN+CN(I)*OMEGA(IW)**NWEXP                                  TF1395
      SUMD=0.0                                                          TF1396
      DO 126 J=1,NDP1                                                   TF1397
      NWEXP=2*(J-1)                                                     TF1398
  126 SUMD=SUMD+CD(J)*OMEGA(IW)**NWEXP                                  TF1399
      XMAG(IW)=ABS(SUMN/SUMD)                                           TF1400
  127 IF(IGRAPH.EQ.1) GO TO 128                                         TF1401
      PHAS1=0.0                                                         TF1402
      GO TO 133                                                         TF1403
  128 PNEV=PAN(1)                                                       TF1404
      DO 129 I=3,NNP1,2                                                 TF1405
  129 PNEV=PNEV+PAN(I)*OMEGA(IW)**(I-1)*(-1)**((I-1)/2)                 TF1406
      PNOD=0.0                                                          TF1407
      DO 130 I=2,NNP1,2                                                 TF1408
  130 PNOD=PNOD+PAN(I)*OMEGA(IW)**(I-1)*(-1)**(I/2+1)                   TF1409
      PDEV=PAD(1)                                                       TF1410
      DO 131 I=3,NDP1,2                                                 TF1411
  131 PDEV=PDEV+PAD(I)*OMEGA(IW)**(I-1)*(-1)**((I-1)/2)                 TF1412
      PDOD=0.0                                                          TF1413
      DO 132 I=2,NDP1,2                                                 TF1414
  132 PDOD=PDOD+PAD(I)*OMEGA(IW)**(I-1)*(-1)**(I/2+1)                   TF1415
      PREAL=PNEV*PDEV+PNOD*PDOD                                         TF1416
      PIMAG=PNOD*PDEV-PNEV*PDOD                                         TF1417
      TVAL=PIMAG/PREAL                                                  TF1418
      PHAS1=ATAN(TVAL)                                                  TF1419
      IF(PREAL.LT.0.) PHAS1=-3.141593+PHAS1                             TF1420
      IF((PREAL.GT.0.).AND.(PIMAG.GT.0.)) PHAS1=-2.0*3.141593+PHAS1     TF1421
      PHSDEG(IW)=(PHAS1*180.0)/3.141593                                 TF1422
  133 IF(NVINPT.GE.1) XMAG(IW)=XMAG(IW)*(VINPUT(KV)**2.0)               TF1423
      IF(MAGSCL.EQ.0) GO TO 134                                         TF1424
```

```
      IF(MAGSCL.EQ.2) GO TO 136                                         TF1425
      XMAG(IW)=SQRT(XMAG(IW))                                           TF1426
      GO TO 136                                                         TF1427
  134 IF((XMAG(IW)-EPSLOG).GE.0.0) GO TO 135                            TF1428
      XMAG(IW)=-1.0E 70                                                 TF1429
      GO TO 140                                                         TF1430
  135 XMAG(IW)=10.0*ALOG10(XMAG(IW))                                    TF1431
  136 IF(PLOTNR.NE.5) GO TO 139                                         TF1432
      XX=XMAG(IW)                                                       TF1433
      OMG=OMEGA(IW)                                                     TF1434
      CALL ABVAR (AL,BE,XX,OMG)                                         TF1435
      IF (KNR.EQ.0) GO TO 137                                           TF1436
      GO TO 91                                                          TF1437
  137 ALPHA(IW)=AL                                                      TF1438
      BETA(IW)=BE                                                       TF1439
  138 KK1=1                                                             TF1440
  139 IF((XMAG(IW).LT.MAGMIN).OR.(XMAG(IW).GT.MAGMAX)) GO TO 140        TF1441
      IP=IP+1                                                           TF1442
      MAXIP(IC)=IP                                                      TF1443
      PHASE(IC,IP)=PHSDEG(IW)                                           TF1444
      X(IP)=(MAGMAX-XMAG(IW))/XSCALE                                    TF1445
      YY(IC,IP)=(OMEGA(IW)-ZMIN)/YSCALE                                 TF1446
      IF(OMGSCL.EQ.0) YY(IC,IP)=(ALOG10(OMEGA(IW))-ZLOGMN)/YSCALE       TF1447
      Y(IP)=YY(IC,IP)                                                   TF1448
  140 IW=IW+1                                                           TF1449
      IF(IW.GT.NRPNTS) GO TO 141                                        TF1450
      IF(NVINPT.GE.1) GO TO 81                                          TF1451
      GO TO 121                                                         TF1452
C****************** WRITE CUT RESULTS ********************************** TF1453
  141 WRITE(6,1)                                                        TF1454
      IF(IWRITE.EQ.0) GO TO 166                                         TF1455
  142 WRITE(6,143) IC                                                   TF1456
  143 FORMAT(/,10X,'THE FOLLOWING POINTS WERE DETERMINED FOR CURVE NUMBE TF1457
     *R',I3,/)                                                          TF1458
  144 FORMAT(10X,'POINT NR',5X,'ALPHA',7X,'BETA',6X,'MAGNITUDE',5X,'OMEG TF1459
     *A',/)                                                             TF1460
  145 FORMAT(14X,I3,1X,1P4E12.3)                                        TF1461
  146 FORMAT(10X,'POINT NR',5X,'ALPHA',7X,'BETA',6X,'MAGNITUDE',5X,'OMEG TF1462
     *A',7X,'PHASE',6X,'VINPUT',/)                                      TF1463
  147 FORMAT(14X,I3,1X,1P6E12.3)                                        TF1464
  148 GO TO (149,151,153,155,157),PLOTNR                                TF1465
  149 WRITE(6,144)                                                      TF1466
      DO 150 KP=1,NRPNTS,IWRITE                                         TF1467
  150 WRITE(6,145) KP,ALPHA(KP),SETBET,XMAG(KP),CURPAR(ID)              TF1468
      GO TO 162                                                         TF1469
  151 WRITE(6,144)                                                      TF1470
      DO 152 KP=1,NRPNTS,IWRITE                                         TF1471
  152 WRITE(6,145) KP,SETALP,BETA(KP),XMAG(KP),CURPAR(ID)               TF1472
```

```
      GO TO 162
  153 WRITE(6,146)
      DO 154 KP=1,NRPNTS,IWRITE
  154 WRITE(6,147) KP,CURPAR(ID),SETBET,XMAG(KP),OMEGA(KP),PHSDEG(KP),VI
     *NPUT(KV)
      GO TO 159
  155 WRITE(6,146)
      DO 156 KP=1,NRPNTS,IWRITE
  156 WRITE(6,147) KP,SETALP,CURPAR(ID),XMAG(KP),OMEGA(KP),PHSDEG(KP),VI
     *NPUT(KV)
      GO TO 159
  157 WRITE(6,146)
      DO 158 KP=1,NRPNTS,IWRITE
  158 WRITE(6,147) KP,ALPHA(KP),BETA(KP),XMAG(KP),OMEGA(KP),PHSDEG(KP),V
     *INPUT(KV)
  159 WRITE(6,160)
  160 FORMAT(/,10X,'THE ORIGIN OF THIS GRAPH HAS BEEN SHIFTED TO THE LOW
     *ER RIGHT HAND CORNER OF THE PAGE.',/,10X,'ROTATE PAGE 90 DEGREES C
     *LOCKWISE FOR STANDARD FORM.',/)
      IF(IGRAPH.EQ.1) WRITE(6,161)
  161 FORMAT(10X,'THE PHASE PLOT MUST ALSO BE ROTATED 90 DEGREES CLOCKWI
     *SE TO PUT IT IN THE STANDARD FORM.',/)
  162 IF(IP.LT.2) GO TO 164
      WRITE(6,163) IP
  163 FORMAT(/,10X,'NUMBER OF POINTS PLOTTED (NUMBER WITHIN SPECIFIED RA
     *NGE):',I4)
      GO TO 166
  164 WRITE(6,165) IC
  165 FORMAT(/,10X,'CURVE NUMBER',I3,' WAS NOT PLOTTED DUE TO AN INSUFFI
     *CIENT NUMBER OF ACCEPTABLE POINTS.',/)
      GO TO 167
C****************** PLOT CURVE ****************************************
  166 IF(IGRAPH.EQ.-1) GO TO 167
      CALL GRPLOT(2)
      IP=0
      IN=1
      IW=1
  167 IF(NVINPT.EQ.0) GO TO 168
      KV=KV+1
      IF(KV.GT.NVINPT) GO TO 168
      GO TO 81
  168 ID=ID+1
      IF(ID.GT.NPCURV) GO TO 169
      GO TO 81
  169 IF(IGRAPH.EQ.-1) GO TO 175
      CALL GRPLOT(3)
      IF((IGRAPH.EQ.0).OR.(PLCTNR.LE.2)) GO TO 175
C****************** PLOT PHASE ****************************************
```

```
      XMAX=PHSMAX                                                       TF1521
      XMIN=PHSMIN                                                       TF1522
      XUPL=PHSUPL                                                       TF1523
      MSCALE=6                                                          TF1524
      DO 170 L=1,14                                                     TF1525
      GTITL1(L)=GTITL4(L)                                               TF1526
      GTITL2(L)=GTITL5(L)                                               TF1527
  170 GTITL3(L)=GTITL6(L)                                               TF1528
      CALL GRPLOT(1)                                                    TF1529
  171 IC=1                                                              TF1530
  172 K=MAXIP(IC)                                                       TF1531
      DO 173 L=1,K                                                      TF1532
      X(L)=(PHSMAX-PHASE(IC,L))/XSCALE                                  TF1533
  173 Y(L)=YY(IC,L)                                                     TF1534
      IP=K                                                              TF1535
      CALL GRPLOT(2)                                                    TF1536
      IC=IC+1                                                           TF1537
      NPHFIN=NPCURV                                                     TF1538
      IF(PLOTNR.EQ.5) NPHFIN=NVINPT                                     TF1539
      IF(IC.GT.NPHFIN) GO TO 174                                        TF1540
      GO TO 172                                                         TF1541
  174 CALL GRPLOT(3)                                                    TF1542
C****************** END RUN ******************************************* TF1543
  175 IRUN=IRUN+1                                                       TF1544
      IF(IRUN.GT.NRUN) GO TO 900                                        TF1545
      GO TO 18                                                          TF1546
  900 CONTINUE                                                          TF1547
      STOP                                                              TF1548
      END                                                               TF1549

      SUBROUTINE GRPLOT(K)                                              TF1550
C                                                                       TF1551
C   THIS SUBROUTINE PROVIDES THE PLOTTING PACKAGE FOR THE PROGRAM. THE  TF1552
C   THE SUBROUTINES CALLED ARE EXPLAINED IN U.S.N. POSTGRADUATE SCHOOL  TF1553
C   COMPUTER FACILITY TECHNICAL NOTE NO. 0211-03.                       TF1554
C                                                                       TF1555
      INTEGER PLOTNR,ZSCALE                                             TF1556
      REAL LGSCAL(70)/4H.001,4H.002,4H.003,4H.004,4H.005,4H.006,4H.007, TF1557
     *4H.008,4H.009,4H0.01,4H0.01,4H0.02,4H0.03,4H0.04,4H0.05,4H0.06,   TF1558
     *4H0.07,4H0.08,4H0.09,4H 0.1,4H 0.1,4H 0.2,4H 0.3,4H 0.4,4H 0.5,   TF1559
     *4H 0.6,4H 0.7,4H 0.8,4H 0.9,4H 1.0,4H 1.0,4H 2.0,4H 3.0,4H 4.0,   TF1560
     *4H 5.0,4H 6.0,4H 7.0,4H 8.0,4H 9.0,4H10.0,4H10.0,4H20.0,4H30.0,   TF1561
     *4H40.0,4H50.0,4H60.0,4H70.0,4H80.0,4H90.0,4H100.,4H100.,4H200.,   TF1562
     *4H300.,4H400.,4H500.,4H600.,4H700.,4H800.,4H900.,4H1000,4H1000,   TF1563
     *4H2000,4H3000,4H4000,4H5000,4H6000,4H7000,4H8000,4H9000,4H    /   TF1564
      REAL*8 ANAME(6)/8HALPHA   ,8HBETA    ,8HOMEGA   ,8HMAG     ,8HMAG( TF1565
     *DB)  ,8HPHASE   /                                                 TF1566
```

```
      DIMENSION  GTITL1(14),GTITL2(14),GTITL3(14),GTITL4(14),GTITL5(14),  TE1567
     *     GTITL6(14)                                                     TE1568
      DIMENSION X(901),Y(901)                                             TE1569
      COMMON MAGSCL,MARVAR,EPSLOG,KNR,KK1,MDATA                           TE1570
      COMMON GTITL1,GTITL2,GTITL3,GTITL4,GTITL5,GTITL6                    TE1571
      COMMON XMAX,XMIN,ZMAX,ZMIN,MSCALE,XSCALE,YSCALE,ZSCALE,ZLOGMN       TE1572
      COMMON X,Y,XUPL,YUPL,XWIDE,YHIGH,PLOTNR,IC,IP                       TE1573
      IF(K-2) 10,30,40                                                    TE1574
   10 CALL PLOTS                                                          TE1575
C****************** LABEL GRAPH *************************************** TE1576
      CALL SYMBOL (0.0,0.50,0.14,GTITL1,0.0,56)                           TE1577
      CALL SYMBOL (0.0,0.25,0.14,GTITL2,0.0,56)                           TE1578
      CALL SYMBOL (0.0,0.0,0.14,GTITL3,0.0,56)                            TE1579
      CALL PLOT (0.0,1.0,-3)                                              TE1580
C****************** DETERMINE SCALING ********************************* TE1581
      IF(ZSCALE.EQ.0) GO TO 12                                            TE1582
      YSCALE=(ZMAX-ZMIN)/YHIGH                                            TE1583
      YINCR=YUPL/YSCALE                                                   TE1584
      GO TO 14                                                            TE1585
   12 IF(ZMIN.LT.1000.0) ZLOGMN=2.0                                       TE1586
      IF(ZMIN.LT.100.0) ZLOGMN=1.0                                        TE1587
      IF(ZMIN.LT.10.0) ZLOGMN=0.0                                         TE1588
      IF(ZMIN.LT.1.0) ZLOGMN=-1.0                                         TE1589
      IF(ZMIN.LT.0.1) ZLOGMN=-2.0                                         TE1590
      IF(ZMIN.LT.0.01) ZLOGMN=-3.0                                        TE1591
      IF(ZMAX.GT.0.01) ZLOGMX=-1.0                                        TE1592
      IF(ZMAX.GT.0.1) ZLOGMX=0.0                                          TE1593
      IF(ZMAX.GT.1.0) ZLOGMX=1.0                                          TE1594
      IF(ZMAX.GT.10.0) ZLOGMX=2.0                                         TE1595
      IF(ZMAX.GT.100.0) ZLOGMX=3.0                                        TE1596
      IF(ZMAX.GT.1000.0) ZLOGMX=4.0                                       TE1597
      ADECAD=ZLOGMX-ZLOGMN                                                TE1598
      NDECAD=ADECAD                                                       TE1599
      YSCALE=ADECAD/YHIGH                                                 TE1600
   14 XSCALE=(XMAX-XMIN)/XWIDE                                            TE1601
      XINCR=XUPL/XSCALE                                                   TE1602
C****************** DRAW HORIZONTAL AXIS MARKERS ********************** TE1603
      NRVLP1=(XMAX-XMIN)/XUPL+1                                           TE1604
      DO 16 LNR=1,NRVLP1                                                  TE1605
      X(1)=(LNR-1)*XINCR                                                  TE1606
      X(2)=X(1)                                                           TE1607
      Y(1)=YHIGH                                                          TE1608
      Y(2)=0.0                                                            TE1609
      CALL LINE (X,Y,2,1,1)                                               TE1610
      ZNUM=XMAX-(LNR-1)*XUPL                                              TE1611
      IF(PLOTNR.LE.2) ZNUM=XMIN+(LNR-1)*XUPL                              TE1612
      XZ=X(1)-0.14                                                        TE1613
      YZ=-0.11                                                            TE1614
```

```
   16 CALL NUMBER (XZ,YZ,0.07,ZNUM,0.0,2)
C****************** LABEL HORIZONTAL AXIS *********************************
      XZ=XWIDE/2.0-0.24
      YZ=-0.29
      LTR=4
      IF(MSCALE.EQ.0) LTR=5
      IF(MSCALE.EQ.6) LTR=6
      CALL SYMBOL (XZ,YZ,0.14,ANAME(LTR),0.0,8)
      IF(ZSCALE.EQ.0) GO TO 20
C****************** DRAW VERTICAL AXIS MARKERS ***************************
      NRHLP1=(ZMAX-ZMIN)/YUPL+1
      DO 18 LNR=1,NRHLP1
      X(1)=0.0
      X(2)=XWIDE
      Y(1)=(LNR-1)*YINCR
      Y(2)=Y(1)
      CALL LINE (X,Y,2,1,1)
      YNUM=ZMIN+(LNR-1)*YUPL
      XZ=X(2)+0.05
      YZ=Y(2)
   18 CALL NUMBER (XZ,YZ,0.07,YNUM,0.0,2)
      GO TO 24
   20 DO 22 INUM=1,NDECAD
      DO 22 JNUM=1,10
      ZNUM=JNUM
      X(1)=0.0
      X(2)=XWIDE
      Y(1)=(ALOG10(ZNUM)+INUM-1)/YSCALE
      Y(2)=Y(1)
      CALL LINE (X,Y,2,1,1)
      IF(JNUM.GT.5) GO TO 22
      LNUM=10.0*ZLOGMN+30.0+(INUM-1)*10.0+JNUM
      XZ=X(2)+0.05
      YZ=Y(2)
      CALL SYMBOL (XZ,YZ,0.07,LGSCAL(LNUM),0.0,4)
   22 CONTINUE
C****************** LABEL VERTICAL AXIS ***********************************
   24 XZ=XWIDE+0.47
      YZ=YHIGH/2.0-0.36
      LTR=3
      IF(PLOTNR.LE.2) LTR=PLOTNR
      CALL SYMBOL (XZ,YZ,0.14,ANAME(LTR),90.0,8)
      GO TO 90
C****************** PLOT CURVES *******************************************
   30 ZIC=IC
      XCNR=X(IP)+0.02
      YCNR=Y(IP)-0.02
      CALL LINE (X,Y,IP,1,1)
```

```
      CALL NUMBER (XCNR,YCNR,0.07,ZIC,0.0,-1)                           TF1663
      GO TO 90                                                          TF1664
C****************** END GRAPH ******************************************TF1665
   40 YUP=YHIGH+4.0                                                     TF1666
      CALL PLOT (0.0,YUP,-3)                                            TF1667
      CALL PLOTE                                                        TF1668
   90 RETURN                                                            TF1669
      END                                                               TF1670


      SUBROUTINE INTTEN (AL,BE,W,X2)                                    TF1671
C                                                                       TF1672
C    THIS SUBROUTINE IS CALLED BY THE ABVAR SUBROUTINE WHEN MDATA =1.   TF1673
C    IT IS NEEDED WHEN A PARAMETER IS A FUNCTION OF AN INTERNAL TRANSFER TF1674
C    FUNCTION. THE SUBROUTINE COMPUTES THE MAGNITUDE OF THE INTERNAL    TF1675
C    TRANSFER FUNCTION AND RETURNS IT TO THE ABVAR SUBROUTINE AS X2. THE TF1676
C    ADDITIONAL DATA CARDS REQUIRED ARE INDICATED BELOW.                TF1677
C                                                                       TF1678
C CARD FORMAT   SYMBOL              SYMBOL USAGE                        TF1679
C  17    8I10   NRNUM2  ORDER OF THE NUMERATOR POLYNOMIAL OF THE INTERNAL TF1680
C                       TRANSFER FUNCTION. (MAX=10)                     TF1681
C               NRDEN2  ORDER OF THE DENOMINATOR POLYNOMIAL OF THE      TF1682
C                       INTERNAL TRANSFER FUNCTION. (MAX=10)            TF1683
C                                                                       TF1684
C  18  8E10.3     AN    NUMERATOR COEFFICIENTS IN ASCENDING ORDER OF    TF1685
C                       S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S TF1686
C                       EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER. TF1687
C                       (1)  *1.0,    *B,       *B**2,       *B**3,     TF1688
C                            *A,      *A*B,     *A*B**2,     *A*B**3.   TF1689
C                       (2)  *A**2,   *A**2*B,  *A**2*B**2,  *A**2*B**3, TF1690
C                            *A**3,   *A**3*B,  *A**3*B**2,  *A**3*B**3. TF1691
C                                                                       TF1692
C  10  8E10.3     AD    DENOMINATOR COEFFICIENTS IN ASCENDING ORDER OF  TF1693
C                       S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S TF1694
C                       EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER. TF1695
C                       SAME FORMAT AS CARD NUMBER 18 (NUMERATOR COEFF). TF1696
C                                                                       TF1697
C****************** PROGRAM FOLLOWS ************************************TF1698
      DIMENSION AA(11,16),AD(11,16),AN(11,16)                           TF1699
      DIMENSION A(11),C(11),CD(11),CN(11)                               TF1700
      COMMON MAGSCL,MABVAR,EPSLOG,KNR,KK1,MDATA                         TF1701
    1 FORMAT(1H1)                                                       TF1702
    2 FORMAT(8I10)                                                      TF1703
    4 FORMAT(10X,8I10)                                                  TF1704
    5 FORMAT(8E10.3)                                                    TF1705
    6 FORMAT(10X,8E10.3)                                                TF1706
      IF(MABVAR.GT.1) GO TO 91                                          TF1707
C****************** READ AND WRITE TRANSFER FUNCTION DECK **************TF1708
```

```
      WRITE(6,1)                                                        TF1709
      READ(5,2) NRNUM2,NRDEN2                                           TF1710
      NNP1=NRNUM2+1                                                     TF1711
      NDP1=NRDEN2+1                                                     TF1712
      DO 12 JN=1,NNP1                                                   TF1713
   12 READ(5,5) (AN(JN,I),I=1,16)                                       TF1714
      DO 14 JD=1,NDP1                                                   TF1715
   14 READ(5,5) (AD(JD,I),I=1,16)                                       TF1716
      WRITE(6,16)                                                       TF1717
   16 FORMAT(10X,'THE INTERNAL TRANSFER FUNCTION DATA FOLLOWS:',/)      TF1718
      WRITE(6,18)                                                       TF1719
   18 FORMAT(16X,'NRNUM2',4X,'NRDEN2')                                  TF1720
      WRITE(6,4) NRNUM2,NRDEN2                                          TF1721
   53 WRITE(6,54)                                                       TF1722
   54 FORMAT(//,10X,'NUMERATOR FUNCTION',/)                             TF1723
   55 FORMAT(3X,'S**',I2,' * (',1P1E10.3,'*1.0         + ',1P1E10.3,'*B TF1724
     *        + ',1P1E10.3,'*B**2        + ',1P1E10.3,'*B**3       + ',/,12X, TF1725
     *1P1E10.3,'*A            + ',1P1E10.3,'*A*B         + ',1P1E10.3,'*A*B** TF1726
     *2      + ',1P1E10.3,'*A*B**3      + ',/,12X,1P1E10.3,'*A**2         + ',1 TF1727
     *P1E10.3,'*A**2*B      + ',1P1E10.3,'*A**2*B**2 + ',1P1E10.3,'*A**2*B TF1728
     ***3 + ',/,12X,1P1E10.3,'*A**3         + ',1P1E10.3,'*A**3*B      + ',1P TF1729
     *1E10.3,'*A**3*B**2 + ',1P1E10.3,'*A**3*B**3)',/)                  TF1730
      DO 56 JN=1,NNP1                                                   TF1731
      JNM1=JN-1                                                         TF1732
   56 WRITE(6,55) JNM1,(AN(JN,I),I=1,16)                                TF1733
      WRITE(6,58)                                                       TF1734
   58 FORMAT(//,10X,'DENOMINATOR FUNCTION',/)                           TF1735
      DO 59 JD=1,NDP1                                                   TF1736
      JDM1=JD-1                                                         TF1737
   59 WRITE(6,55) JDM1,(AD(JD,I),I=1,16)                                TF1738
      MABVAR=MABVAR+1                                                   TF1739
      WRITE(6,1)                                                        TF1740
C***************** CALCULATE MAGNITUDE ********************************* TF1741
   91 IKOUNT=1                                                          TF1742
      N=NNP1                                                            TF1743
      DO 92 I=1,N                                                       TF1744
      DO 92 J=1,16                                                      TF1745
   92 AA(I,J)=AN(I,J)                                                   TF1746
   93 DO 94 I=1,N                                                       TF1747
   94 A(I)=AA(I,1)+AA(I,2)*BE+AA(I,3)*BE**2+AA(I,4)*BE**3+AA(I,5)*AL+AA( TF1748
     *I,6)*AL*BE+AA(I,7)*AL*BE**2+AA(I,8)*AL*BE**3+AA(I,9)*AL**2+AA(I,10 TF1749
     *)*AL**2*BE+AA(I,11)*AL**2*BE**2+AA(I,12)*AL**2*BE**3+AA(I,13)*AL** TF1750
     *3+AA(I,14)*AL**3*BE+AA(I,15)*AL**3*BE**2+AA(I,16)*AL**3*BE**3    TF1751
      IF(N.EQ.11) GO TO 96                                              TF1752
      NP1=N+1                                                           TF1753
      DO 95 I=NP1,11                                                    TF1754
   95 A(I)=0.0                                                          TF1755
   96 GO TO (107,106,105,104,103,102,101,100,99,98,97),N                TF1756
```

```
 97 C(11)=A(11)*A(11)                                                        TF1757
 98 C(10)=A(10)*A(10)-2.0*A(9)*A(1)                                          TF1758
 99 C(9)=A(9)*A(9)+2.0*(A(7)*A(11)-A(8)*A(10))                               TF1759
100 C(8)=A(8)*A(8)-2.0*(A(5)*A(11)-A(6)*A(10)+A(7)*A(9))                     TF1760
101 C(7)=A(7)*A(7)+2.0*(A(3)*A(11)-A(4)*A(10)+A(5)*A(9)-A(6)*A(8))           TF1761
102 C(6)=A(6)*A(6)-2.0*(A(1)*A(11)-A(2)*A(10)+A(3)*A(9)-A(4)*A(8)+A(5)       TF1762
   **A(7))                                                                   TF1763
103 C(5)=A(5)*A(5)+2.0*(A(1)*A(9)-A(2)*A(8)+A(3)*A(7)-A(4)*A(6))             TF1764
104 C(4)=A(4)*A(4)-2.0*(A(1)*A(7)-A(2)*A(6)+A(3)*A(5))                       TF1765
105 C(3)=A(3)*A(3)+2.0*(A(1)*A(5)-A(2)*A(4))                                 TF1766
106 C(2)=A(2)*A(2)-2.0*A(1)*A(3)                                             TF1767
107 C(1)=A(1)*A(1)                                                           TF1768
    IF(IKOUNT.EQ.2) GO TO 110                                                TF1769
    DO 108 IG=1,N                                                            TF1770
108 CN(IG)=C(IG)                                                             TF1771
    N=NDP1                                                                   TF1772
    IKOUNT=2                                                                 TF1773
    DO 109 I=1,N                                                             TF1774
    DO 109 J=1,16                                                            TF1775
109 AA(I,J)=AD(I,J)                                                          TF1776
    GO TO 93                                                                 TF1777
110 DO 111 IG=1,N                                                            TF1778
111 CD(IG)=C(IG)                                                             TF1779
112 SUMN=0.0                                                                 TF1780
    DO 113 I=1,NNP1                                                          TF1781
    NWEXP=2*(I-1)                                                            TF1782
113 SUMN=SUMN+CN(I)*W**NWEXP                                                 TF1783
    SUMD=0.0                                                                 TF1784
    DO 114 J=1,NDP1                                                          TF1785
    NWEXP=2*(J-1)                                                            TF1786
114 SUMD=SUMD+CD(J)*W**NWEXP                                                 TF1787
    X2=ABS(SUMN/SUMD)                                                        TF1788
    IF(MAGSCL.EQ.2) GO TO 120                                                TF1789
    IF(MAGSCL.EQ.0) GO TO 116                                                TF1790
    X2=SQRT(X2)                                                              TF1791
    GO TO 120                                                                TF1792
116 IF((X2-EPSLOG).GE.0.0) GO TO 118                                         TF1793
    X2=-1000.0                                                               TF1794
    GO TO 120                                                                TF1795
118 X2=10.0*ALOG10(X2)                                                       TF1796
120 RETURN                                                                   TF1797
    END                                                                      TF1798

    SUBROUTINE ARVAR (AL,BE,XX,OMG)                                          TF1799
C                                                                            TF1800
C   THIS SUBROUTINE IS USED TO DESCRIBE THE NONLINEAR PARAMETER CHAR-        TF1801
C   ACTERISTICS WHEN PLOTNP=5. ADD THE NECESSARY FORTRAN STATEMENTS          TF1802
```

```
C     TO REFLECT THE DESIRED RELATIONSHIPS (X=MAGNITUDE IN MAGSCL UNITS).   TF1803
C                                                                           TF1804
      COMMON MAGSCL,MABVAR,EPSLOG,KNR,KK1,MDATA                             TF1805
      A=AL                                                                  TF1806
      B=BE                                                                  TF1807
      X=XX                                                                  TF1808
      W=OMG                                                                 TF1809
      EPSA=1.0E-04                                                          TF1810
      EPSB=1.0E-04                                                          TF1811
      ANEW=A                                                                TF1812
      BNEW=B                                                                TF1813
      IF(MDATA.NE.1) GO TO 10                                               TF1814
      CALL INTTFN (AL,BE,W,X2)                                              TF1815
   10 CONTINUE                                                              TF1816
      ADEL=ABS(ANEW-A)                                                      TF1817
      BDEL=ABS(BNEW-B)                                                      TF1818
      KNR=0                                                                 TF1819
      IF((ADEL.GE.EPSA).OR.(BDEL.GE.EPSB)) KNR=1                            TF1820
      AL=ANEW                                                               TF1821
      BE=BNEW                                                               TF1822
      KK1=KK1+1                                                             TF1823
   20 RETURN                                                                TF1824
      END                                                                   TF1825
```

PARTF2  PROGRAM

```
C
C    PARTF2,PARAMETER PLANE TRANSFER FUNCTION PLOTTING ROUTINE NUMBER 2,
C    ALPHA,BETA VERSUS OMEGA AND ALPHA VERSUS BETA. PROGRAM INPUT IS THE
C    CIRCUIT TRANSFER FUNCTION IN THE S-PLANE, T(S). THE PROGRAM COMPUTES
C    T(JW) AND SOLVES FOR ALPHA OR BETA. T(S) IS LIMITED TO TENTH ORDER.
C    THE MAXIMUM EXPONENT VALUE FOR ALPHA AND BETA IS THREE (SEE FORMAT
C    CARD NUMBER EIGHT).
C
C
C  CARD FORMAT   SYMBOL              SYMBOL USAGE
C
C    1     8I1C    NRUN    NUMBER OF RUNS DESIRED. EACH RUN MUST HAVE A FULL
C                          SET OF DATA CARDS.
C
C    2     8I1C   PLOTNR   PLOT NUMBER (Y-AXIS VS X-AXIS)
C                          1=ALPHA VS OMEGA,BETA FIXED,MAGNITUDE PARAMETER
C                          2=BETA VS OMEGA,ALPHA FIXED,MAGNITUDE PARAMETER
C                          3=ALPHA VS BETA,OMEGA FIXED,MAGNITUDE PARAMETER
C                          4=ALPHA VS BETA,MAGNITUDE FIXED,OMEGA PARAMETER
C                 NORNUM   ORDER OF NUMERATOR POLYNOMIAL (MAX=10).
C                 NORDEN   ORDER OF DENOMINATOR POLYNOMIAL (MAX=10).
C                 NPCURV   NUMBER OF PARAMETER VALUES (1-16).
C                 NRPNTS   NUMBER OF COMPUTATIONS BETWEEN MAX AND MINVALUES.
C                          IT EQUALS THE NUMBER OF POINTS PLOTTED PER CURVE
C                          (1-900). IF SET=0, NRPNTS=900.
C                 IWRITE   NUMBER OF COMPUTATIONS BETWEEN PRINTOUTS (1-900).
C                          IF SET=0, NO COMPUTATION RESULTS ARE PRINTED.
C
C    3     8I1C   IGRAPH   GRAPH ORDER.
C                          -1= NO GRAPHS DESIRED.
C                           0= GRAPHS ARE DESIRED.
C                 ALPSCL   ALPHA OUTPUT AXIS SCALE CODE. USED WITH PLOTNR 1,
C                          3,4.
C                          0=LOG SCALE.
C                          1=LINEAR SCALE.
C                 BETSCL   BETA OUTPUT AXIS SCALE CODE. USED WITH PLOTNR 2,
C                          3,4.
C                          0=LOG SCALE.
C                          1=LINEAR SCALE.
C                 MAGSCL   MAGNITUDE PARAMETER SCALE CODE.
C                          0=DB VALUES.
```

```
C                         1=ACTUAL VALUES.                                   TF2041
C                         2=SQUARED VALUES.                                  TF2042
C               OMGSCL    OMEGA OUTPUT AXIS SCALE CODE.                      TF2043
C                         0=LOG SCALE.                                       TF2044
C                         1=LINEAR SCALE.                                    TF2045
C                                                                            TF2046
C     4   8E10.3   YHIGH  HEIGHT OF THE VERTICAL AXIS, Y-AXIS, OF THE GRAPH  TF2047
C                         IN INCHES. FOR TOTAL PAGE HEIGHT ADD 1.0 INCHES    TF2048
C                         FOR GRAPH SCALES, AXIS LABEL AND GRAPH TITLING.    TF2049
C                  XWIDE  WIDTH OF THE HORIZONTAL AXIS, X-AXIS, OF THE       TF2050
C                         GRAPH IN INCHES. FOR TOTAL PAGE WIDTH ADD 0.5      TF2051
C                         INCHES FOR GRAPH SCALES AND AXIS LABEL. TOTAL      TF2052
C                         WIDTH IS LIMITED BY PAPER SIZE, USUALLY 9.0 INCHES TF2053
C                   YUPL  NUMBER OF UNITS PER LINE ON THE VERTICAL SCALE.    TF2054
C                         IF SET=0, IT WILL BE SCALED AUTOMATICALLY TO       TF2055
C                         (YMAX-YMIN)/YHIGH. IT IS NOT USED WITH LOG SCALES. TF2056
C                   XUPL  NUMBER OF UNITS PER LINE ON THE HORIZONTAL SCALE.  TF2057
C                         IF SET=0, IT WILL BE SCALED AUTOMATICALLY TO       TF2058
C                         (XMAX-XMIN)/XWIDE. IT IS NOT USED WITH LOG SCALES. TF2059
C                                                                            TF2060
C     5   8E10.3  ALPMIN  MINIMUM VALUE FOR ALPHA.                           TF2061
C                 ALPMAX  MAXIMUM VALUE FOR ALPHA.                           TF2062
C                 BETMIN  MINIMUM VALUE FOR BETA.                            TF2063
C                 BETMAX  MAXIMUM VALUE FOR BETA.                            TF2064
C                 MAGMIN  MINIMUM VALUE FOR MAGNITUDE. SEE MAGSCL.           TF2065
C                 MAGMAX  MAXIMUM VALUE FOR MAGNITUDE. SEE MAGSCL.           TF2066
C                 OMGMIN  MINIMUM VALUE FOR OMEGA.                           TF2067
C                 OMGMAX  MAXIMUM VALUE FOR OMEGA.                           TF2068
C    NOTE: IF AXIS SCALE CODE IS LOG, MIN.GE.0.001, MAX.LE.10000.0.          TF2069
C                                                                            TF2070
C     6   8E10.3  SETALP  VALUE OF ALPHA WHEN USED AS A CONSTANT.            TF2071
C                 SETBET  VALUE OF BETA WHEN USED AS A CONSTANT.             TF2072
C                 SETMAG  VALUE OF MAGNITUDE WHEN USED AS A CONSTANT. SEE    TF2073
C                         MAGSCL.                                            TF2074
C                 SETOMG  VALUE OF OMEGA WHEN USED AS A CONSTANT.            TF2075
C                                                                            TF2076
C     7   8E10.3  CURPAR  PARAMETER VALUES (1-16). IF NPCURV .GT. 8          TF2077
C                         USE TWO CARDS.                                     TF2078
C                                                                            TF2079
C     8   8E10.3    XC    NUMERATOR COEFFICIENTS IN ASCENDING ORDER OF       TF2080
C                         S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S  TF2081
C                         EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER.  TF2082
C                         (1)  *1.0,    *B,       *B**2,       *B**3,        TF2083
C                              *A,      *A*B,     *A*B**2,     *A*B**3.      TF2084
C                         (2)  *A**2,   *A**2*B,  *A**2*B**2,  *A**2*B**3,   TF2085
C                              *A**3,   *A**3*B,  *A**3*B**2,  *A**3*B**3.   TF2086
C                                                                            TF2087
C     9   8E10.3    XC    DENOMINATOR COEFFICIENTS IN ASCENDING ORDER OF     TF2088
```

```
C                     S EXPONENTS. TWO CARDS ARE USED FOR EACH OF THE S    TF2089
C                     EXPONENTS FROM S**0 THROUGH THE POLYNOMIAL ORDER.    TF2090
C                     SAME FORMAT AS CARD NUMBER 8 (NUMERATOR COEFF).      TF2091
C                                                                          TF2092
C  10   14A4   GTITL1  GRAPH TITLE LINE NUMBER ONE.                        TF2093
C                                                                          TF2094
C  11   14A4   GTITL2  GRAPH TITLE LINE NUMBER TWO.                        TF2095
C                                                                          TF2096
C  12   14A4   GTITL3  GRAPH TITLE LINE NUMBER THREE.                      TF2097
C                                                                          TF2098
C   NOTE: OMIT CARDS 10-12 IF IGRAPH=-1.                                   TF2099
C                                                                          TF2100
C                                                                          TF2101
C****************** PROGRAM FOLLOWS ************************************  TF2102
      INTEGER PLOTNR,ALPSCL,BETSCL,OMGSCL,ZSCALE                           TF2103
      REAL MAGMAX,MAGMIN                                                   TF2104
      DIMENSION XI(6),XR(6),XN(7),XRO(6),XIO(6),CURPAR(16)                 TF2105
      DIMENSION  GTITL1(14),GTITL2(14),GTITL3(14)                          TF2106
      DIMENSION X(901),Y(901)                                              TF2107
      DIMENSION XP(900,6),YP(900,6)                                        TF2108
      DIMENSION XK(2,11,7),XC(2,11,16),XM(2,11,11,7),XMP(2,11,11,7)        TF2109
      COMMON GTITL1,GTITL2,GTITL3                                          TF2110
      COMMON XMAX,XMIN,ZMAX,ZMIN,NSCALE,XSCALE,YSCALE,ZSCALE,ZLOGMN,YLOG   TF2111
     *MN                                                                   TF2112
      COMMON X,Y,XLPL,YUPL,XWIDE,YHIGH,PLOTNR,ID,NRPP                      TF2113
    1 FORMAT(1H1)                                                          TF2114
    3 FORMAT(10X,'ID=',I3,2X,'IW=',I3,2X,'IER=',I3)                        TF2115
    5 FORMAT(14A4)                                                         TF2116
    6 FORMAT(10X,14A4)                                                     TF2117
    7 FORMAT(8I10)                                                         TF2118
    8 FORMAT(10X,8I10)                                                     TF2119
    9 FORMAT(8F10.3)                                                       TF2120
   10 FORMAT(10X,1P8E12.3)                                                 TF2121
C****************** READ IN DATA **************************************  TF2122
      READ(5,7) NRUN                                                       TF2123
      IRUN=1                                                               TF2124
      IER=9                                                                TF2125
   11 READ(5,7) PLOTNR,NCRNUM,NORDEN,NPCURV,NRPNTS,IWRITE                  TF2126
      NNP1=NCRNUM+1                                                        TF2127
      NDP1=NCRDEN+1                                                        TF2128
      NDM2=NDP1-2                                                          TF2129
      IF(NRPNTS.EQ.0) NRPNTS=900                                           TF2130
      NRPTM1=NRPNTS-1                                                      TF2131
      READ(5,7) IGRAPH,ALPSCL,BETSCL,MAGSCL,OMGSCL                         TF2132
      READ(5,9) YHIGH,XWIDE,YUPL,XUPL                                      TF2133
      READ(5,9) ALPMIN,ALPMAX,BETMIN,BETMAX,MAGMIN,MAGMAX,OMGMIN,OMGMAX    TF2134
      READ(5,9) SETALP,SETBET,SETMAG,SETOMG                                TF2135
      READ(5,9) (CURPAR(I),I=1,NPCURV)                                     TF2136
```

```
      DO 12 I=1,2
      DO 12 J=1,11
      DO 12 K=1,16
   12 XC(I,J,K)=0.0
      DO 13 J=1,NNP1
   13 READ(5,9) (XC(1,J,K),K=1,16)
      DO 14 J=1,NDP1
   14 READ(5,9) (XC(2,J,K),K=1,16)
      DO 15 I=1,2
      DO 15 J=1,NDP1
      DO 15 K=1,7
   15 XK(I,J,K)=0.0
      IF(IGRAPH.EQ.-1) GO TO 16
      READ(5,5) (GTITL1(I),I=1,14)
      READ(5,5) (GTITL2(I),I=1,14)
      READ(5,5) (GTITL3(I),I=1,14)
C****************** WRITE INPUT DATA *************************************
   16 WRITE(6,1)
      IF(IRUN.GT.1) GO TO 18
      WRITE(6,17) NRUN
   17 FORMAT(14X,'NRUN=',I3)
   18 WRITE(6,19)
   19 FORMAT(/,16X,'PLOTNR',4X,'NORNUM',4X,'NORDEN',4X,'NPCURV',4X,'NRPN
     *TS',4X,'IWRITE')
      WRITE(6,8) PLOTNR,NORNUM,NORDEN,NPCURV,NRPNTS,IWRITE
      WRITE(6,20)
   20 FORMAT(/,16X,'IGRAPH',4X,'ALPSCL',4X,'BETSCL',4X,'MAGSCL',4X,'OMGS
     *CL')
      WRITE(6,8) IGRAPH,ALPSCL,BETSCL,MAGSCL,OMGSCL
      IF(YUPL.NE.0.0) GO TO 21
      YUPL=(ALPMAX-ALPMIN)/YHIGH
      IF(PLOTNR.EQ.2) YUPL=(BETMAX-BETMIN)/YHIGH
   21 IF(XUPL.NE.0.0) GO TO 22
      XUPL=(BETMAX-BETMIN)/XWIDE
      IF(PLOTNR.LE.2) XUPL=(OMGMAX-OMGMIN)/XWIDE
   22 WRITE(6,23)
   23 FORMAT(/,15X,'YHIGH',7X,'XWIDE',7X,'YUPL',9X,'XUPL')
      WRITE(6,10) YHIGH,XWIDE,YUPL,XUPL
      WRITE(6,24)
   24 FORMAT(/,14X,'ALPMIN',6X,'ALPMAX',6X,'BETMIN',6X,'BETMAX',6X,'MAGM
     *IN',6X,'MAGMAX',6X,'OMGMIN',6X,'OMGMAX')
      WRITE(6,10)ALPMIN,ALPMAX,BETMIN,BETMAX,MAGMIN,MAGMAX,OMGMIN,OMGMAX
      WRITE(6,25)
   25 FORMAT(/,14X,'SETALP',6X,'SETBET',6X,'SETMAG',6X,'SETOMG')
      WRITE(6,10) SETALP,SETBET,SETMAG,SETOMG
      IF(PLOTNR.EQ.4) GO TO 28
   26 WRITE(6,27)
   27 FORMAT(//,10X,'THE VALUES FOR THE MAGNITUDE PARAMETER CURVES ARE',
```

```
     */)
      GO TO 30
   28 WRITE(6,29)
   29 FORMAT(//,10X,'THE VALUES FOR THE OMEGA PARAMETER CURVES ARE',/)
   30 WRITE(6,10) (CURPAR(I),I=1,NPCURV)
   32 WRITE(6,33)
   33 FORMAT(//,10X,'NUMERATOR FUNCTION',/)
   34 FORMAT(3X,'S**',I2,' * (',1P1E10.3,'*1.0         + ',1P1E10.3,'*B
     *        + ',1P1E10.3,'*B**2        + ',1P1E10.3,'*B**3          + ',/,12X,
     *1P1E10.3,'*A            + ',1P1E10.3,'*A*B          + ',1P1E10.3,'*A*B**
     *2     + ',1P1E10.3,'*A*B**3      + ',/,12X,1P1E10.3,'*A**2         + ',1
     *P1E10.3,'*A**2*B     + ',1P1E10.3,'*A**2*B**2 + ',1P1E10.3,'*A**2*B
     ***3 + ',/,12X,1P1E10.3,'*A**3          + ',1P1E10.3,'*A**3*B      + ',1P
     *1E10.3,'*A**3*B**2 + ',1P1E10.3,'*A**3*B**3)',/)
      DO 35 J=1,NNF1
      JM1=J-1
   35 WRITE(6,34) JM1,(XC(1,J,K),K=1,16)
      WRITE(6,36)
   36 FORMAT(//,10X,'DENOMINATOR FUNCTION',/)
      DO 37 J=1,NDF1
      JM1=J-1
   37 WRITE(6,34) JM1,(XC(2,J,K),K=1,16)
      IF(IGRAPH.EQ.-1) GO TO 39
      WRITE(6,38)
   38 FORMAT(//,10X,'THE GRAPH WILL BE TITLED',/)
      WRITE(6,6) (GTITL1(IT),IT=1,14)
      WRITE(6,6) (GTITL2(IT),IT=1,14)
      WRITE(6,6) (GTITL3(IT),IT=1,14)
      WRITE(6,1)
C****************** SET UP GRAPH SCALES ***********************************
   39 IF(PLOTNP.EQ.2) GO TO 40
      ZMAX=ALPMAX
      ZMIN=ALPMIN
      ZSCALE=ALPSCL
      GO TO 41
   40 ZMAX=BETMAX
      ZMIN=BETMIN
      ZSCALE=BETSCL
   41 IF(PLOTNR.LE.2) GO TO 42
      XMAX=BETMAX
      XMIN=BETMIN
      NSCALE=BETSCL
      GO TO 43
   42 XMAX=OMGMAX
      XMIN=OMGMIN
      NSCALE=OMGSCL
   43 CALL GRPLOT (1)
C****************** COLLAPSE COEFFICIENT MATRICIES ***********************
```

```
45 ID=1
46 DO 47 L=1,6
   XRO(L)=0.0050010
47 XIO(L)=0.0100010
   IW=1
   IP=1
   WRITE(6,1)
   WRITE(6,48) ID
48 FORMAT(/,1X,'THE FOLLOWING POINTS WERE DETERMINED FOR CURVE NR',I3
  *,/)
   IF(PLOTNR-2) 49,51,53
49 WRITE(6,50)
50 FORMAT(1X,'POINT NR',6X,'BETA',6X,'MAGNITUDE',5X,'OMEGA',7X,'ALPHA
  *  (ROOT OF POLYNOMIAL-IMAG PART LISTED UNDER REAL PART)')
   GO TO 55
51 WRITE(6,52)
52 FORMAT(1X,'POINT NR',5X,'ALPHA',5X,'MAGNITUDE',5X,'OMEGA',7X,'BETA
  *  (ROOT OF POLYNOMIAL-IMAG PART LISTED UNDER REAL PART)')
   GO TO 55
53 WRITE(6,54)
54 FORMAT(1X,'POINT NR',5X,'OMEGA',5X,'MAGNITUDE',5X,'BETA',8X,'ALPHA
  *  (ROOT OF POLYNOMIAL-IMAG PART LISTED UNDER REAL PART)')
55 IF(ID.EQ.1) GO TO 56
   GO TO (73,73,58,58), PLOTNR
56 GO TO (57,65,58,58),PLOTNR
57 BE=SETBET
   SETVAL=BE
   GO TO 63
58 BINC=(BETMAX-BETMIN)/NRPTM1
   IF(BETSCL.EQ.0) BINC=(ALOG10(BETMAX)-ALOG10(BETMIN))/NRPTM1
   IF(PLOTNR.EQ.4) GO TO 59
   W=SETOMG
   SETVAL=W
   XMA=CURPAR(ID)
   IF(MAGSCL.EQ.1) XMA=XMA**2
   IF(MAGSCL.EQ.0) XMA=10.0**(XMA/10.0)
   GO TO 60
59 XMA=SETMAG
   IF(MAGSCL.EQ.1) XMA=XMA**2
   IF(MAGSCL.EQ.0) XMA=10.0**(XMA/10.0)
   SETVAL=XMA
   W=CURPAR(ID)
60 IF(BETSCL.EQ.0) GO TO 61
   BE=BETMIN+(IW-1)*BINC
   GO TO 62
61 BEL=ALOG10(BETMIN)+(IW-1)*BINC
   BE=10.0**BEL
62 XH=BE
```

```
 63 DO 64 I=1,2
    DO 64 J=1,NDP1
    DO 64 K=1,4
    K4=4*K
    K4M2=K4-2
    K4M3=K4-3
    XK(I,J,K)=XC(I,J,K4M3)
    DO 64 K1=K4M2,K4
 64 XK(I,J,K)=(BE**(K1-K4M3))*XC(I,J,K1)+XK(I,J,K)
    PARMIN=ALPMIN
    PARMAX=ALPMAX
    GO TO 68
 65 AL=SETALP
    SETVAL=AL
    PARMIN=BETMIN
    PARMAX=BETMAX
 66 DO 67 I=1,2
    DO 67 J=1,NDP1
    DO 67 K=1,4
    KP4=K+4
    XK(I,J,K)=XC(I,J,K)
    KEXP=0
    DO 67 K1=KP4,16,4
    KEXP=KEXP+1
 67 XK(I,J,K)=(AL**KEXP)*XC(I,J,K1)+XK(I,J,K)
C****************** FORM COEFFICIENT MATRIX PRODUCTS ******************
 68 DO 72 I=1,2
    DO 70 J=1,NDP1
    DO 70 JJ=J,NDP1,2
    DO 69 K=1,7
 69 XM(I,J,JJ,K)=0.0
    DO 70 K1=1,4
    DO 70 K2=1,4
    K=K1+K2-1
 70 XM(I,J,JJ,K)=XK(I,J,K1)*XK(I,JJ,K2)+XM(I,J,JJ,K)
    DO 72 J=1,NDM2
    JP2=J+2
    KS=1
    DO 72 JJ=JP2,NDP1,2
    DO 71 K=1,7
 71 XM(I,J,JJ,K)=(-1.0**KS)*2.0*XM(I,J,JJ,K)
 72 KS=KS+1
C****************** SET MAGNITUDE AND OMEGA ****************************
    IF((PLOTNR.EQ.3).OR.(PLOTNR.EQ.4)) GO TO 78
 73 XMA=CURPAR(IC)
    IF(MAGSCL.EQ.1) XMA=XMA**2
    IF(MAGSCL.EQ.0) XMA=10.0**(XMA/10.0)
 74 WINC=(OMGMAX-OMGMIN)/NRPTM1
```

```
      IF(OMGSCL.EQ.0) WINC=(ALOG10(OMGMAX)-ALOG10(OMGMIN))/NRPTM1
   75 IF(OMGSCL.EQ.0) GO TO 76
      W=OMGMIN+(IW-1)*WINC
      GO TO 77
   76 WL=ALOG10(OMGMIN)+(IW-1)*WINC
      W=10.0**WL
   77 XH=W
C******************* SET UP POLYNOMIAL AND SOLVE FOR ROOTS **************
   78 DO 79 I=1,2
      DO 79 J=1,NDP1
      DO 79 JJ=J,NDP1,2
      XE=(J-1)+(JJ-1)
      DO 79 K=1,7
   79 XMP(I,J,JJ,K)=(W**XE)*XM(I,J,JJ,K)
      IF(XMA.EQ.1.0) XMA=0.99
   80 DO 81 K=1,7
      XN(K)=0.0
      DO 81 J=1,NDP1
      DO 81 JJ=J,NDP1,2
   81 XN(K)=XMP(1,J,JJ,K)-XMA*XMP(2,J,JJ,K)+XN(K)
      DO 82 I=1,7
   82 IF(ABS(XN(I)).LT.1.0E-20) XN(I)=0.0
      KPDIM=7
   83 IF(XN(KPDIM).NE.0.0) GO TO  84
      KPDIM=KPDIM-1
      GO TO  83
   84 KPORD=KPDIM-1
      DO 85 L=1,6
      XR(L)=0.0
   85 XI(L)=0.0
      GO TO (93,86,87,88,88,88,88,110), KPDIM
   86 XR(1)=-XN(1)/XN(2)
      GO TO 91
   87 CALL POLRT2(XN,XR,XI)
      GO TO 91
   88 CALL POLRT3 (XN,COF,KPORD,XR,XI,XRO,XIO,IER)
      IF(IER.EQ.0) GO TO 89
      WRITE(6,3) ID,IW,IER
   89 DO 90 L=1,KPORD
      XRO(L)=-XI(L)/10.0
   90 XIO(L)=-XR(L)/10.0
   91 DO 92 I=1,KPORD
      IF(ABS(XR(I)).LE.1.0E-50) XR(I)=0.0
   92 IF(ABS(XI(I)).LE.1.0E-50) XI(I)=0.0
   93 IF(IW.EQ.1) GO TO 96
      IWK=IW-1
   94 IF(IWK-IWRITE) 101,96,95
   95 IWK=IWK-IWRITE
```

```
      GO TO 94
   96 IF(KPDIM.EQ.1) GO TO 99
      WRITE(6,97) IW,SETVAL,CURPAR(ID),XH,(XR(K),K=1,KPORD)
   97 FORMAT(4X,I3,1P9F12.3)
      WRITE(6,98) (XI(K),K=1,KPORD)
   98 FORMAT(43X,1P6F12.3)
      GO TO 110
   99 WRITE(6,100) IW,XN(1)
  100 FORMAT(/,10X,'ITERATION',I4,' RESULTED IN A DEGENERATE POLYNOMIAL
     *OF ORDER ZERO. CONSTANT=',1P1E12.3,/)
      GO TO 110
C****************** ABSTRACT USEABLE ROOTS *****************************
  101 IPK=0
      DO 102 K=1,KPORD
      IF(XI(K).NE.0.0) GO TO 102
      IF((XR(K).LT.PARMIN).OR.(XR(K).GT.PARMAX)) GO TO 102
      IPK=1
  102 CONTINUE
      IF(IPK.EQ.0) GO TO 110
      DO 103 K=1,KPORD
  103 IF((XR(K).LT.PARMIN).OR.(XR(K).GT.PARMAX)) XR(K)=1.0E50
  104 K=1
  105 IF(XR(K+1).GE.XR(K)) GO TO 106
      XX=XR(K)
      XR(K)=XR(K+1)
      XR(K+1)=XX
      GO TO 104
  106 K=K+1
      IF(K.EQ.KPORD) GO TO 107
      GO TO 105
  107 DO 109 K=1,KPORD
      YP(IP,K)=XR(K)
      IF(PLOTNR.GT.2) GO TO 108
      XP(IP,K)=W
      GO TO 109
  108 XP(IP,K)=RE
  109 CONTINUE
      IP=IP+1
  110 IW=IW+1
      IF(IW.GT.NRPNTS) GO TO 111
      GO TO (75,75,60,60), PLOTNR
C****************** GRAPH DATA ******************************************
  111 IPM1=IP-1
      WRITE(6,112) IPM1,ID
  112 FORMAT(10X,I4,' ACCEPTABLE POINTS WERE FOUND FOR CURVE NUMBER',I3,
     */)
      IF(IGRAPH.EQ.-1) GO TO 132
      IF(IPM1.GE.2) GO TO 115
```

```
113 WRITE(6,114) ID
114 FORMAT(10X,'CURVE NUMBER',I3,' WAS NOT PLOTTED SINCE LESS THAN 2 A
   *CCEPTABLE POINTS WERE FOUND.',/)
    GO TO 130
115 IF(ZSCALE.EQ.0) GO TO 117
    DO 116 IP=1,IPM1
116 Y(IP)=(YP(IP,1)-ZMIN)/YSCALE
    GO TO 119
117 DO 118 IP=1,IPM1
118 Y(IP)=(ALOG10(YP(IP,1))-YLOGMN)/YSCALE
119 IF(NSCALE.EQ.0) GO TO 121
    DO 120 IP=1,IPM1
120 X(IP)=(XP(IP,1)-XMIN)/XSCALE
    GO TO 123
121 DO 122 IP=1,IPM1
122 X(IP)=(ALOG10(XP(IP,1))-ZLOGMN)/XSCALE
    NRPP=IPM1
123 CALL GRPLOT(2)
    IZ=2
124 IQ=1
    IZNR=0
    YVALMX=1.0E50
    DO 125 IP=1,IPM1
125 IF(YP(IP,IZ).LT.YVALMX) IZNR=1
    IF(IZNR.EQ.0) GO TO 130
    DO 128 IP=1,IPM1
    IZP=IZ
126 IF(YP(IP,IZP).GE.YVALMX) GO TO 127
    X(IQ)=X(IP)
    Y(IQ)=(YP(IP,IZP)-ZMIN)/YSCALE
    IF(ZSCALE.EQ.0) Y(IQ)=(ALOG10(YP(IP,IZP))-YLOGMN)/YSCALE
    GO TO 128
127 IZP=IZP-1
    IF(IZP.EQ.0) GO TO 128
    GO TO 127
128 IQ=IQ+1
    NRPP=IQ-1
    CALL GRPLOT(2)
129 IZ=IZ+1
    IF(IZ.EQ.KPDIM) GO TO 130
    GO TO 124
130 ID=ID+1
    IF(ID.GT.NPCURV) GO TO 131
    GO TO 46
131 IF(IGRAPH.EQ.-1) GO TO 132
    CALL GRPLOT (3)
132 IRUN=IRUN+1
    IF(IRUN.GT.NRUN) GO TO 900
```

```
      GO TO 11                                                          TF2473
  900 CONTINUE                                                          TF2474
      STOP                                                              TF2475
      END                                                               TF2476


      SUBROUTINE GRPLOT(K)                                              TF2477
C                                                                       TF2478
C    THIS SUBROUTINE PROVIDES THE PLOTTING PACKAGE FOR THE PROGRAM. THE TF2479
C    THE SUBROUTINES CALLED ARE EXPLAINED IN U.S.N. POSTGRADUATE SCHOOL TF2480
C    COMPUTER FACILITY TECHNICAL NOTE NO. 0211-03.                      TF2481
C                                                                       TF2482
      INTEGER PLOTNR,ZSCALE                                             TF2483
      REAL LGSCAL(70)/4H.001,4H.002,4H.003,4H.004,4H.005,4H.006,4H.007, TF2484
     *4H.008,4H.009,4H0.01,4H0.01,4H0.02,4H0.03,4H0.04,4H0.05,4H0.05,   TF2485
     *4H0.07,4H0.08,4H0.09,4H 0.1,4H 0.1,4H 0.2,4H 0.3,4H 0.4,4H 0.5,   TF2486
     *4H 0.6,4H 0.7,4H 0.8,4H 0.9,4H 1.0,4H 1.0,4H 2.0,4H 3.0,4H 4.0,   TF2487
     *4H 5.0,4H 6.0,4H 7.0,4H 8.0,4H 9.0,4H10.0,4H10.0,4H20.0,4H30.0,   TF2488
     *4H40.0,4H50.0,4H60.0,4H70.0,4H80.0,4H90.0,4H100.,4H100.,4H200.,   TF2489
     *4H300.,4H400.,4H500.,4H600.,4H700.,4H800.,4H900.,4H1000,4H1000,   TF2490
     *4H2000,4H3000,4H4000,4H5000,4H6000,4H7000,4H8000,4H9000,4H    /   TF2491
      REAL*8 ANAME(3)/8HALPHA   ,8HBETA    ,8HOMEGA   /                 TF2492
      DIMENSION  GTITL1(14),GTITL2(14),GTITL3(14)                       TF2493
      DIMENSION X(901),Y(901)                                           TF2494
      COMMON GTITL1,GTITL2,GTITL3                                       TF2495
      COMMON XMAX,XMIN,ZMAX,ZMIN,NSCALE,XSCALE,YSCALE,ZSCALE,ZLOGMN,YLOG TF2496
     *MN                                                                TF2497
      COMMON X,Y,XLPL,YUPL,XWIDE,YHIGH,PLOTNR,ID,NR2P                   TF2498
      IF(K-2) 10,30,40                                                  TF2499
   10 CALL PLOTS                                                        TF2500
C****************** LABEL GRAPH ****************************************TF2501
      CALL SYMBOL (0.0,0.50,0.14,GTITL1,0.0,56)                         TF2502
      CALL SYMBOL (0.0,0.25,0.14,GTITL2,0.0,56)                         TF2503
      CALL SYMBOL (0.0,0.0,0.14,GTITL3,0.0,56)                          TF2504
      CALL PLOT (0.0,1.0,-3)                                            TF2505
C****************** DETERMINE SCALING **********************************TF2506
      ILOG=1                                                            TF2507
      IF(ZSCALE.EQ.0) GO TO 12                                          TF2508
      YSCALE=(ZMAX-ZMIN)/YHIGH                                          TF2509
      YINCR=YUPL/YSCALE                                                 TF2510
      GO TO 13                                                          TF2511
   12 IF(ZMIN.LT.1000.0) ZLOGMN=2.0                                     TF2512
      IF(ZMIN.LT.100.0) ZLOGMN=1.0                                      TF2513
      IF(ZMIN.LT.10.0) ZLOGMN=0.0                                       TF2514
      IF(ZMIN.LT.1.0) ZLOGMN=-1.0                                       TF2515
      IF(ZMIN.LT.0.1) ZLOGMN=-2.0                                       TF2516
      IF(ZMIN.LT.0.01) ZLOGMN=-3.0                                      TF2517
      IF(ZMAX.GT.0.01) ZLOGMX=-1.0                                      TF2518
```

```
      IF(ZMAX.GT.0.1) ZLOGMX=0.0
      IF(ZMAX.GT.1.0) ZLOGMX=1.0
      IF(ZMAX.GT.10.0) ZLOGMX=2.0
      IF(ZMAX.GT.100.0) ZLOGMX=3.0
      IF(ZMAX.GT.1000.0) ZLOGMX=4.0
      ADECAD=ZLOGMX-ZLOGMN
      IF(ILOG.EQ.0) GO TO 17
      NDECAD=ADECAD
      YLOGMN=ZLOGMN
      YSCALE=ADECAD/YHIGH
   13 IF(NSCALE.EQ.0) GO TO 16
      XSCALE=(XMAX-XMIN)/XWIDE
      XINCR=XUPL/XSCALE
C****************** DRAW HORIZONTAL AXIS MARKERS ***********************
      NRVLP1=(XMAX-XMIN)/XUPL+1
      DO 15 LNR=1,NRVLP1
      X(1)=(LNR-1)*XINCR
      X(2)=X(1)
      Y(1)=YHIGH
      Y(2)=0.0
      CALL LINE (X,Y,2,1,1)
      ZNUM=XMAX-(LNR-1)*XUPL
      IF(PLOTNR.LE.2) ZNUM=XMIN+(LNR-1)*XUPL
      XZ=X(1)-0.14
      YZ=-0.11
   15 CALL NUMBER (XZ,YZ,0.07,ZNUM,0.0,2)
      GO TO 19
   16 ILOG=0
      TZMIN=ZMIN
      ZMIN=XMIN
      ZMAX=XMAX
      GO TO 12
   17 MDECAD=ADECAD
      XSCALE=ADECAD/XWIDE
      ZMIN=TZMIN
      DO 18 INUM=1,MDECAD
      DO 18 JNUM=1,10
      ZNUM=JNUM
      X(1)=(ALOG10(ZNUM)+INUM-1)/XSCALE
      X(2)=X(1)
      Y(1)=0.0
      Y(2)=YHIGH
      CALL LINE (X,Y,2,1,1)
      IF(JNUM.NE.1) GO TO 18
      LNUM=10.0*ZLOGMN+30.0+(INUM-1)*10.0+JNUM
      XZ=X(2)-0.14
      YZ=-0.11
      CALL SYMBOL (XZ,YZ,0.07,LGSCAL(LNUM),0.0,4)
```

```
   18 CONTINUE                                                              TF2567
C****************** LABEL HORIZONTAL AXIS ******************************   TF2568
   19 XZ=XWIDE/2.0-0.24                                                     TF2569
      YZ=-0.29                                                              TF2570
      LTR=3                                                                 TF2571
      IF(PLOTNR.GT.2) LTR=2                                                 TF2572
      CALL SYMBOL (XZ,YZ,0.14,ANAME(LTR),0.0,8)                             TF2573
C****************** DRAW VERTICAL AXIS MARKERS ************************    TF2574
      IF(ZSCALE.EQ.0) GO TO 21                                              TF2575
      NRHLP1=(ZMAX-ZMIN)/YUPL+1                                             TF2576
      DO 20 LNR=1,NRHLP1                                                    TF2577
      X(1)=0.0                                                              TF2578
      X(2)=XWIDE                                                            TF2579
      Y(1)=(LNR-1)*YINCR                                                    TF2580
      Y(2)=Y(1)                                                             TF2581
      CALL LINE (X,Y,2,1,1)                                                 TF2582
      YNUM=ZMIN+(LNR-1)*YUPL                                                TF2583
      XZ=X(2)+0.05                                                          TF2584
      YZ=Y(2)                                                               TF2585
   20 CALL NUMBER (XZ,YZ,0.07,YNUM,0.0,2)                                   TF2586
      GO TO 24                                                              TF2587
   21 DO 22 INUM=1,NDECAD                                                   TF2588
      DO 22 JNUM=1,10                                                       TF2589
      ZNUM=JNUM                                                             TF2590
      X(1)=0.0                                                              TF2591
      X(2)=XWIDE                                                            TF2592
      Y(1)=(ALOG10(ZNUM)+INUM-1)/YSCALE                                     TF2593
      Y(2)=Y(1)                                                             TF2594
      CALL LINE (X,Y,2,1,1)                                                 TF2595
      IF(JNUM.GT.5) GO TO 22                                                TF2596
      LNUM=10.0*YLOGMN+30.0+(INUM-1)*10.0+JNUM                              TF2597
      XZ=X(2)+0.05                                                          TF2598
      YZ=Y(2)                                                               TF2599
      CALL SYMBOL (XZ,YZ,0.07,LGSCAL(LNUM),0.0,4)                           TF2600
   22 CONTINUE                                                              TF2601
C****************** LABEL VERTICAL AXIS ********************************   TF2602
   24 XZ=XWIDE+0.47                                                         TF2603
      YZ=YHIGH/2.0-0.36                                                     TF2604
      LTR=1                                                                 TF2605
      IF(PLOTNR.EQ.2) LTR=2                                                 TF2606
      CALL SYMBOL (XZ,YZ,0.14,ANAME(LTR),90.0,8)                            TF2607
      GO TO 90                                                              TF2608
C****************** PLOT CURVES ****************************************   TF2609
   30 ZIC=ID                                                                TF2610
      XCNR=X(NRPP)+0.02                                                     TF2611
      YCNR=Y(NRPP)-0.02                                                     TF2612
      CALL LINE (X,Y,NRPP,1,1)                                              TF2613
      CALL NUMBER (XCNR,YCNR,0.07,ZIC,0.0,-1)                               TF2614
```

```
      GO TO 90
C****************** END GRAPH *********************************************
   40 YUP=YHIGH+4.0
      CALL PLOT (0.0,YUP,-3)
      CALL PLOTE
   90 RETURN
      END


      SUBROUTINE POLRT2(XN,XR,XI)
C
C   THIS SUBROUTINE IS USED TO SOLVE FOR THE ROOTS WHEN THE POLYNOMIAL
C   IS OF SECOND ORDER.
C
      DIMENSION XN(3),XR(2),XI(2)
      XR(1)=-XN(2)/(2.0*XN(3))
      XR(2)=XR(1)
      XI(1)=0.0
      XI(2)=0.0
      XRB1=XN(1)/XN(3)
      DISC=XR(1)**2-XRB1
      IF(DISC) 10,30,20
   10 XI(1)=SQRT(-DISC)
      XI(2)=-XI(1)
      GO TO 30
   20 SDISC=SQRT(DISC)
      XR(1)=XR(1)+SDISC
      XR(2)=XR(1)-SDISC
      IF(DISC.GT.100.0) GO TO 22
      GO TO 30
   22 XRA1=ABS(XR(1))
      XRA2=ABS(XR(2))
      IF(XRA1.LT.XRA2) GO TO 24
      XR(2)=XRB1/XR(1)
      GO TO 30
   24 XR(1)=XRB1/XR(2)
   30 RETURN
      END


      SUBROUTINE POLRT3(XCOF,COF,M,ROOTR,ROOTI,XRO,XIO,IER)
C
C   THIS SUBROUTINE IS USED TO SOLVE FOR THE ROOTS WHEN THE POLYNOMIAL
C   IS OF ORDER THREE OR GREATER.
C
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1),XRO(1),XIO(1)
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     *DX,DY,TEMP,ALPHA
```

```
C
C     ..................................................................
C
      CALL ERRSET (207,300,300,2,1,208)
      IFIT=0
      N=M
      IER=0
      IF(XCOF(N+1))10,25,10
   10 IF(N) 15,15,32
C****************** SET ERROR CODES ***********************************
   15 IER=1
   20 RETURN
   25 IER=4
      GO TO 20
   31 IER=3
      GO TO 20
   30 IER=2
      GO TO 20
   32 IF(N-36) 35,35,30
C****************** SET VALUES AND COUNTER ****************************
   35 NX=N
      NXX=N+1
      N2=1
      KJ1 = N+1
      DO 40 L=1,KJ1
      MT=KJ1-L+1
   40 COF(MT)=XCOF(L)
   42 INO=1
      GO TO 46
   45 INO=INO+1
   46 XO=DBLE(XRC(INO))
      YO=DBLE(XIC(INO))
   49 IN=0
   50 X=XO
      XO=-10.0*YO
      YO=-10.0*X
      IF(XO.LE.1.0E7) GO TO 54
      XO=0.00500101
      YO=0.01000101
      GO TO 49
   51 ICT=ICT+1
      IF(ICT-500) 60,52,52
   52 IF(IFIT) 100,53,100
   53 IF(IN-5) 50,31,31
C****************** SET X AND Y ***************************************
   54 X=XO
      Y=YO
      IN=IN+1
```

```
      GO TO 59
   55 IFIT=1
      XPR=X
      YPR=Y
C****************** EVALUATE POLYNOMIAL AND DERIVATIVES ****************
   59 ICT=0
   60 UX=0.0
      UY=0.0
      V =0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,130,65
   65 DO 70 I=1,N
      L =N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      FI=I
      UX=UX+FI*XT*TEMP
      UY=UY-FI*YT*TEMP
      XT=XT2
   70 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
   75 DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
   78 IF(DABS(DY)+DABS(DX)-1.0D-05) 100,51,51
  100 DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
  105 COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 120,55,120
  110 IF(IFIT) 115,50,115
  115 X=XPR
      Y=YPR
  120 IFIT=0
  122 IF(DABS(Y)-1.0D-4*DABS(X)) 135,125,125
  125 ALPHA=X+X
      SUMSQ=X*X+Y*Y
```

```
      N=N-2                                                               TE2755
      GO TO 140                                                           TE2756
  130 X=0.0                                                               TE2757
      NX=NX-1                                                             TE2758
      NXX=NXX-1                                                           TE2759
  135 Y=0.0                                                               TE2760
      SUMSQ=0.0                                                           TE2761
      ALPHA=X                                                             TE2762
      N=N-1                                                               TE2763
  140 COF(2)=COF(2)+ALPHA*COF(1)                                          TE2764
  145 DO 150 L=2,N                                                        TE2765
  150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)                       TE2766
  155 ROOTI(N2)=Y                                                         TE2767
      ROOTR(N2)=X                                                         TE2768
      N2=N2+1                                                             TE2769
      IF(SUMSQ) 160,165,160                                               TE2770
  160 Y=-Y                                                                TE2771
      SUMSQ=0.0                                                           TE2772
      GO TO 155                                                           TE2773
  165 IF(N) 20,20,45                                                      TE2774
  200 RETURN                                                              TE2775
      END                                                                 TE2776
```

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13. ABSTRACT

A parameter-plane method, based upon a quasi-frozen assumption, for the design of electrical filters containing slowly varying nonlinear elements is presented together with examples of its application to the lowpass, highpass, bandpass and band-reject filter types. The nonlinear magnitude and frequency characteristics are presented in the form of parameter-plane graphs which allow value judgements to be made concerning the choice of nonlinear elements used in the filter design. The computer programs used to generate the graphs are also included.

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| COMPUTER GRAPHICS | | | | | | |